Disliking to disagree

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Technical Appendix [for online publication]

Appendix I: Preliminaries

Throughout the proofs we use the following notation for the perceived disagreement under equilibrium of type $X = \{D0, D1, FD, ND\}$ given the disclosed information $d = \{0, 1, \varnothing\}$:

$$\Delta^X(d) = \left| E_R[\widetilde{\beta}_S(\sigma)|d] - \widetilde{\beta}_R(d) \right|.$$

Note that trivially, $\Delta^X(1)$ and $\Delta^X(0)$ are actually independent of X while $\Delta^X(\varnothing)$ is not, so that we typically omit the superscript in the first two cases. Note also that the notation is slightly abusive in the sense that it does not make explicit that $\Delta^X(d)$ depends on β_S, β_R. Besides, it is convenient to denote the highest and the lowest prior belief as, respectively

$$\begin{aligned} \overline{\beta} &= \max\{\beta_S, \beta_R\}, \\ \underline{\beta} &= \min\{\beta_S, \beta_R\}. \end{aligned}$$

We now characterize equilibrium posterior beliefs (obtained by applying Bayes' rule) that shall be used in checking incentives in different putative equilibria. Note that in any equilibrium

$$\begin{aligned} \widetilde{\beta}_i(1) &= \frac{\Pr[\sigma = 1|\omega = 1]\beta_i}{\Pr[\sigma = 1|\omega = 1]\beta_i + \Pr[\sigma = 1|\omega = 0](1 - \beta_i)} = \frac{p\beta_i}{p\beta_i + (1-p)(1-\beta_i)}, \\ \widetilde{\beta}_i(0) &= \frac{\Pr[\sigma = 0|\omega = 1]\beta_i}{\Pr[\sigma = 0|\omega = 1]\beta_i + \Pr[\sigma = 0|\omega = 0](1 - \beta_i)} = \frac{(1-p)\beta_i}{(1-p)\beta_i + p(1-\beta_i)}, \end{aligned}$$

In a ND-equilibrium, $\widetilde{\beta}_R^{ND}(\varnothing) = \beta_R$ and

$$\begin{aligned}
E_R^{ND}[\widetilde{\beta}_S|\varnothing] &= \Pr[\sigma = 0|d = \varnothing, ND]\widetilde{\beta}_S(0) + \Pr[\sigma = 1|d = \varnothing, ND]\widetilde{\beta}_S(1) \\
&\quad + \Pr[\sigma = \varnothing|d = \varnothing, ND]\beta_S \\
&= \varphi((1-p)\beta_R + p(1-\beta_R))\widetilde{\beta}_S(0) \\
&\quad + \varphi(p\beta_R + (1-p)(1-\beta_R))\widetilde{\beta}_S(1) \\
&\quad + (1-\varphi)\,\beta_S.
\end{aligned}$$

In a D1-equilibrium:

$$\begin{aligned}
\widetilde{\beta}_R^{D1}(\varnothing) &= \Pr[\sigma = 0|d = \varnothing, D1]\widetilde{\beta}_R(0) + \Pr[\sigma = \varnothing|d = \varnothing, D1]\beta_R \\
&= \frac{\Pr[\sigma = 0]}{\Pr[\sigma = 0] + \Pr[\sigma = \varnothing]}\widetilde{\beta}_R(0) + \frac{\Pr[\sigma = \varnothing]}{\Pr[\sigma = 0] + \Pr[\sigma = \varnothing]}\beta_R \\
&= \frac{\varphi((1-p)\beta_R + p(1-\beta_R))}{\varphi((1-p)\beta_R + p(1-\beta_R)) + (1-\varphi)}\widetilde{\beta}_R(0) \\
&\quad + \frac{1-\varphi}{\varphi((1-p)\beta_R + p(1-\beta_R)) + (1-\varphi)}\beta_R, \\
E_R^{D1}[\widetilde{\beta}_S|\varnothing] &= \Pr[\sigma = 0|d = \varnothing, D1]\widetilde{\beta}_S(0) + \Pr[\sigma = \varnothing|d = \varnothing, D1]\beta_S \\
&= \frac{\varphi((1-p)\beta_R + p(1-\beta_R))}{\varphi((1-p)\beta_R + p(1-\beta_R)) + (1-\varphi)}\widetilde{\beta}_S(0) \\
&\quad + \frac{1-\varphi}{\varphi((1-p)\beta_R + p(1-\beta_R)) + (1-\varphi)}\beta_S.
\end{aligned}$$

In a D0-equilibrium:

$$\begin{aligned}
\widetilde{\beta}_R^{D0}(\varnothing) &= \Pr[\sigma = 1|d = \varnothing, D0]\widetilde{\beta}_R(1) + \Pr[\sigma = \varnothing|d = \varnothing, D0]\beta_R \\
&= \frac{\Pr[\sigma = 1]}{\Pr[\sigma = 1] + \Pr[\sigma = \varnothing]}\widetilde{\beta}_R(1) + \frac{\Pr[\sigma = \varnothing]}{\Pr[\sigma = 1] + \Pr[\sigma = \varnothing]}\beta_R \\
&= \frac{\varphi(p\beta_R + (1-p)(1-\beta_R))}{\varphi(p\beta_R + (1-p)(1-\beta_R)) + (1-\varphi)}\widetilde{\beta}_R(1) \\
&\quad + \frac{1-\varphi}{\varphi(p\beta_R + (1-p)(1-\beta_R)) + (1-\varphi)}\beta_R,
\end{aligned}$$

$$
\begin{aligned}
E_R^{D0}[\widetilde{\beta}_S|\varnothing] &= \Pr[\sigma = 1|d = \varnothing, D0]\widetilde{\beta}_S(1) + \Pr[\sigma = \varnothing|d = \varnothing, D0]\beta_S \\
&= \frac{\varphi(p\beta_R + (1-p)(1-\beta_R))}{\varphi(p\beta_R + (1-p)(1-\beta_R)) + (1-\varphi)}\widetilde{\beta}_S(1) \\
&\quad + \frac{1-\varphi}{\varphi(p\beta_R + (1-p)(1-\beta_R)) + (1-\varphi)}\beta_S.
\end{aligned}
$$

Appendix II: Proposition 1 and Corollary 1

Proof of Proposition 1

Proposition **1** follows from a set of Lemmas, which are stated and proved in what follows.

Lemma II.A *Assume* $\beta_S \neq \beta_R$. *Then,* $E_S[\Delta^{FD}] < |\beta_S - \beta_R|$. *I.e. under full disclosure, from S's ex ante perspective, the expected value of R's ex post perceived disagreement is strictly reduced relative to prior disagreement.*

Proof. Assume without loss of generality that $\beta_S > \beta_R$. Then, the difference between prior disagreement and S's ex ante expected value of ex post perceived disagreement under full disclosure is

$$
\begin{aligned}
(\beta_S - \beta_R) - E_S[\Delta^{FD}] &= (\beta_S - \beta_R) \\
&\quad -\varphi(\beta_S p + (1-\beta_S)(1-p))\Delta(1) \\
&\quad -\varphi(\beta_S(1-p) + (1-\beta_S)p)\Delta(0) \\
&\quad -(1-\varphi)(\beta_S - \beta_R) \\
&= (\beta_S - \beta_R) - \varphi(\beta_S p + (1-\beta_S)(1-p)) \\
&\quad \times \left(\frac{\beta_S p}{\beta_S p + (1-\beta_S)(1-p)} - \frac{\beta_R p}{\beta_R p + (1-\beta_R)(1-p)} \right) \\
&\quad -\varphi(\beta_S(1-p) + (1-\beta_S)p) \\
&\quad \times \left(\frac{\beta_S(1-p)}{\beta_S(1-p) + (1-\beta_S)p} - \frac{\beta_R(1-p)}{\beta_R(1-p) + (1-\beta_R)p} \right) \\
&\quad -(1-\varphi)(\beta_S - \beta_R)
\end{aligned}
$$

$$= \varphi \frac{(\beta_S - \beta_R)(1-\beta_R)\beta_R(2p-1)^2}{(1-p+\beta_R(2p-1))(\beta_R + p(1-2\beta_R))} > 0.$$

Hence, from S's ex ante perspective, the full disclosure strategy will on average reduce perceived disagreement relative to prior disagreement. ■

Lemma II.B *Unless $\beta_S = \beta_R$, there exists no ND-equilibrium (where S omits to disclose all signals).*

Proof.

Consider $\beta_S > \beta_R$. Assume by contradiction that there exists an ND-equilibrium. Then, R's perceived disagreement conditional on no disclosure is

$$\begin{aligned} \Delta^{ND}(\varnothing) &= \left| E_R^{ND}[\widetilde{\beta}_S(\sigma)|\varnothing] - \widetilde{\beta}_R^{ND}(\varnothing) \right| \\ &= \sum_{\sigma \in \{0,1,\varnothing\}} \left(P(\sigma \,|\beta_R) \left(\widetilde{\beta}_S(\sigma) - \widetilde{\beta}_R(\sigma) \right) \right), \end{aligned}$$

where $P(\sigma \,|\beta_R)$ is the ex ante probability attributed by R to signal $\sigma \in \{0,1,\varnothing\}$, where $\varnothing$ stands for no signal. If $\beta_S \neq \beta_R$, then Lemma II.A implies

$$\min\{\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0), \widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)\} < \beta_S - \beta_R.$$

Consequently, for $\beta_S \neq \beta_R$ we have (given that $\widetilde{\beta}_S(\varnothing) - \widetilde{\beta}_R(\varnothing) = \beta_S - \beta_R$)

$$\sum_{\sigma \in \{0,1,\varnothing\}} \left(P(\sigma \,|\beta_R) \left(\widetilde{\beta}_S(\sigma) - \widetilde{\beta}_R(\sigma) \right) \right) > \min\{\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0), \widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)\}.$$

Hence, in a putative ND-equilibrium, for some $\sigma \in \{0,1\}$ S would have a strict incentive to deviate by disclosing σ. The case of $\beta_R > \beta_S$ proceeds analogously.

Finally, if $\beta_S = \beta_R$, we trivially have

$$\sum_{\sigma \in \{0,1,\varnothing\}} \left(P(\sigma \,|\beta_R) \left(\widetilde{\beta}_S(\sigma) - \widetilde{\beta}_R(\sigma) \right) \right) = 0 = \widetilde{\beta}_S(0) - \widetilde{\beta}_R(0) = \widetilde{\beta}_S(1) - \widetilde{\beta}_R(1),$$

so that S has no strict incentive to deviate from the equilibrium strategy given $\sigma \in \{0,1\}$. ■

Lemma II.C *If $\beta_S = \beta_R$, then any disclosure strategy is an equilibrium disclosure strategy.*

Proof. Let $\beta_S = \beta_R$ and fix any disclosure strategy $\widetilde{D}$ of S. Denote by δ_R^η the probability assigned by R to $\sigma = \eta$ given $d = \varnothing$, R's prior being β_R. I.e., formally, $\delta_R^\eta = \Pr[\sigma = \eta | d = \varnothing, \beta_R]$. We have

$$\begin{aligned}
\Delta^{\widetilde{D}}(\varnothing) &= \left| E_R^{\widetilde{D}}[\widetilde{\beta}_S|\varnothing] - \widetilde{\beta}_R^{\widetilde{D}}(\varnothing) \right| \\
&= \left| \begin{array}{c} \delta_R^0 \widetilde{\beta}_S(0) + \delta_R^1 \widetilde{\beta}_S(1) + (1 - \delta_R^0 - \delta_R^1)\beta_S \\ -\delta_R^0 \widetilde{\beta}_R(0) - \delta_R^1 \widetilde{\beta}_R(1) - (1 - \delta_R^0 - \delta_R^1)\beta_R \end{array} \right| \\
&= \left| \begin{array}{c} \delta_R^0(\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)) + \delta_R^1(\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)) \\ +(1 - \delta_R^0 - \delta_R^1)(\beta_S - \beta_R) \end{array} \right| = 0, \qquad (4)
\end{aligned}$$

where the last equality is due to $\beta_S = \beta_R$. Hence, S will be indifferent between disclosure of any signal (leading to 0 posterior disagreement) and non-disclosure. Consequently, the specified disclosure strategy constitutes an equilibrium. ■

Lemma II.D *Let $\beta_S \neq \beta_R$. D0 exists if and only if $\beta_S \leq \beta_S^*(\beta_R)$.*

Proof. The D0-equilibrium exists if and only if the following S's incentive constraints are satisfied:

$$\Delta^{D0}(0) \leq \Delta^{D0}(\varnothing) \leq \Delta^{D0}(1). \qquad (5)$$

Using (4), the second incentive constraint simplifies to (denoting again $\delta_R^\eta = \Pr[\sigma = \eta | d = \varnothing, \beta_R]$):

$$\begin{aligned}
\Delta^{D0}(\varnothing) - \Delta^{D0}(1) &\leq 0 \Leftrightarrow \qquad (6) \\
\left| \delta_R^1(\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)) + (1 - \delta_R^1)(\beta_S - \beta_R) \right| - \left| \widetilde{\beta}_S(1) - \widetilde{\beta}_R(1) \right| &\leq 0 \Leftrightarrow \qquad (7) \\
(1 - \delta_R^1)\left(\overline{\beta} - \underline{\beta} - \frac{\overline{\beta}p}{\overline{\beta}p + (1 - \overline{\beta})(1 - p)} + \frac{\underline{\beta}p}{\underline{\beta}p + (1 - \underline{\beta})(1 - p)} \right) &\leq 0 \Leftrightarrow \qquad (8)
\end{aligned}$$

$$\left(1 - \frac{\varphi(\underline{\beta}p + (1-\underline{\beta})(1-p))}{\varphi(\underline{\beta}p + (1-\underline{\beta})(1-p)) + (1-\varphi)}\right)$$
$$\times\left(\overline{\beta} - \underline{\beta} - \frac{\overline{\beta}p}{\overline{\beta}p + (1-\overline{\beta})(1-p)} + \frac{\underline{\beta}p}{\underline{\beta}p + (1-\underline{\beta})(1-p)}\right) \leq 0 \Leftrightarrow \quad (9)$$
$$(\overline{\beta} - \underline{\beta})(2p-1)(1-\varphi)$$
$$\times\frac{\overline{\beta}(1-p+\underline{\beta}(2p-1)) - (1-p)(1-\underline{\beta})}{(1-p+\overline{\beta}(2p-1))(1-p+\underline{\beta}(2p-1))(1-\varphi(p-\underline{\beta}(2p-1))} \leq 0. \quad (10)$$

On the left-hand side of the last inequality, all terms are always positive except for the numerator, which is increasing in both $\overline{\beta}$ and $\underline{\beta}$ and is equal to 0 if and only if

$$\overline{\beta} = \frac{(1-\underline{\beta})(1-p)}{1-p+\underline{\beta}(2p-1)} \Leftrightarrow \underline{\beta} = \frac{(1-\overline{\beta})(1-p)}{1-p+\overline{\beta}(2p-1)}.$$

Thus, independently of whether $\beta_S = \overline{\beta}$ or $\beta_S = \underline{\beta}$ (i.e. of whether $\beta_S > \beta_R$ or $\beta_S < \beta_R$) we have

$$\Delta^{D0}(\varnothing) - \Delta^{D0}(1) \leq 0 \text{ if and only if } \beta_S \leq \frac{(1-\beta_R)(1-p)}{1-p+\beta_R(2p-1)} = \beta_S^*(\beta_R). \quad (11)$$

Note further that $\Delta^{D0}(\varnothing) - \Delta^{D0}(1) \leq 0$ implies $\Delta^{D0}(0) \leq \Delta^{D0}(\varnothing)$. Indeed, otherwise we would have $\Delta^{D0}(\varnothing) \leq \min\{\Delta^{D0}(0), \Delta^{D0}(1)\}$, i.e. it would hold true that

$$\delta_R^1|\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)| + (1-\delta_R^1)|\beta_S - \beta_R| \leq \min\left\{\left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right|, \left|\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)\right|\right\}.$$

This, in turn, yields

$$|\beta_S - \beta_R| \leq |\widetilde{\beta}_S(\eta) - \widetilde{\beta}_R(\eta)|, \forall \eta \in \{0,1\}.$$

The latter would imply that (a putative) full disclosure equilibrium does not yield an expected value of ex post perceived disagreement that is strictly smaller than prior disagreement. But this contradicts Lemma II.A. Thus, (5) holds if and only if $\beta_S \leq \beta_S^*(\beta_R)$. ■

Lemma II.E *Let $\beta_S \neq \beta_R$. D1 exists if and only if $\beta_S \geq \beta_S^{**}(\beta_R)$.*

Proof. The D1-equilibrium exists if and only if the following S's incentive constraints are satisfied:

$$\Delta^{D1}(1) \leq \Delta^{D1}(\varnothing) \leq \Delta^{D1}(0). \tag{12}$$

Using (4), the second incentive constraint simplifies to (denoting again $\delta_R^\eta = \Pr[\sigma = \eta | d = \varnothing, \beta_R]$):

$$\begin{aligned}
\Delta^{D1}(\varnothing) - \Delta^{D1}(0) &\leq 0 \Leftrightarrow && (13)\\
\left|\delta_R^0(\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)) + (1-\delta_R^0)(\beta_S - \beta_R)\right| - \left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right| &\leq 0 \Leftrightarrow && (14)\\
(1-\delta_R^0)\left(\overline{\beta} - \underline{\beta} - \frac{\overline{\beta}(1-p)}{\overline{\beta}(1-p)+(1-\overline{\beta})p} + \frac{\underline{\beta}(1-p)}{\underline{\beta}(1-p)+(1-\underline{\beta})p}\right) &\leq 0 \Leftrightarrow && (15)\\
\left(1 - \frac{\varphi(\underline{\beta}(1-p)+(1-\underline{\beta})p)}{\varphi(\underline{\beta}(1-p)+(1-\underline{\beta})p)+(1-\varphi)}\right) & && \\
\times\left(\overline{\beta} - \underline{\beta} - \frac{\overline{\beta}(1-p)}{\overline{\beta}(1-p)+(1-\overline{\beta})p} + \frac{\underline{\beta}(1-p)}{\underline{\beta}(1-p)+(1-\underline{\beta})p}\right) &\leq 0 \Leftrightarrow && (16)\\
(\overline{\beta} - \underline{\beta})(2p-1)(1-\varphi) & && \\
\times\frac{\overline{\beta}(\underline{\beta}(2p-1)-p)+p(1-\underline{\beta})}{(p-\overline{\beta}(2p-1))(p-\underline{\beta}(2p-1))(1-\varphi(1-p+\underline{\beta}(2p-1))} &\leq 0. && (17)
\end{aligned}$$

On the left-hand side of the last inequality, all terms are always positive except for the numerator, which is decreasing in both $\overline{\beta}$ and $\underline{\beta}$ and is equal to 0 if and only if

$$\overline{\beta} = \frac{p(1-\underline{\beta})}{\underline{\beta}+p(1-2\underline{\beta})} \Leftrightarrow \underline{\beta} = \frac{p(1-\overline{\beta})}{\overline{\beta}+p(1-2\overline{\beta})}.$$

Thus, independently of whether $\beta_S = \overline{\beta}$ or $\beta_S = \underline{\beta}$ (i.e. of whether $\beta_S > \beta_R$ or $\beta_S < \beta_R$) we have

$$\Delta^{D1}(\varnothing) - \Delta^{D1}(0) \leq 0 \text{ if and only if } \beta_S \geq \frac{p(1-\beta_R)}{\beta_R + p(1-2\beta_R)} = \beta_S^{**}(\beta_R). \tag{18}$$

Note further that $\Delta^{D1}(\varnothing) - \Delta^{D1}(0) \leq 0$ implies $\Delta^{D1}(1) \leq \Delta^{D1}(\varnothing)$ by the same argument as in the proof of Lemma II.D. Thus, (12) holds if and only if $\beta_S \geq \beta_S^{**}(\beta_R)$.

■

Lemma II.F Let $\beta_S \neq \beta_R$. *FD exists if and only if* $\beta_S \in [\beta_S^*(\beta_R), \beta_S^{**}(\beta_R)]$.

Proof. The FD-equilibrium exists if and only if the following S's incentive constraints are satisfied:

$$|\beta_S - \beta_R| \geq \Delta^{FD}(1), \tag{19}$$

$$|\beta_S - \beta_R| \geq \Delta^{FD}(0). \tag{20}$$

Note that the reverse inequality to (19) holds under the same conditions as (8), which is turn is equivalent to (6). Hence, by the proof of Lemma II.D $|\beta_S - \beta_R| \leq \Delta^{FD}(1)$ iff $\beta_S \leq \beta_S^*(\beta_R)$ (with $|\beta_S - \beta_R| = \Delta^{FD}(1)$ iff $\beta_S = \beta_S^*(\beta_R)$). Consequently, (19) holds if and only if $\beta_S \geq \beta_S^*(\beta_R)$. Analogously, from the proof of Lemma II.E we obtain that (20) holds if and only if $\beta_S \leq \beta_S^{**}(\beta_R)$. Hence, both constraints hold simultaneously if and only if $\beta_S \in [\beta_S^*(\beta_R), \beta_S^{**}(\beta_R)]$. ■

Lemma II.G *Let* $\beta_S \neq \beta_R$. *Mixed strategy equilibria exist if and only if* $\beta_S \in \{\beta_S^*(\beta_R), \beta_S^{**}(\beta_R)\}$.

Proof. First, if $\beta_S \neq \beta_R$, there cannot be an equilibrium in which S's disclosure strategy (call this strategy M) specifies omitting to disclose with a non-degenerate probability after both signals 0 and 1. Indeed, by the same arguments as in the proof of Lemma II.B (using the same notation), one can show that in such an equilibrium (call it an M-equilibrium), it will be true that

$$\Delta^M(\varnothing) = \sum_\sigma P(\sigma\,|\beta_R)|\beta_S(\sigma) - \beta_R(\sigma)| > \min\{\left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right|, \left|\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)\right|\}.$$

Hence, after some signal $\widetilde{\sigma} \in \{0, 1\}$, ex post perceived disagreement will be strictly smaller than $\Delta^M(\varnothing)$. But given this, S would deviate to disclosing for sure when holding signal $\widetilde{\sigma}$.

Consider now the remaining case of an equilibrium in which S's disclosure strategy (call this strategy $\widetilde{M}$) specifies mixing between disclosure and non-disclosure only for one signal $\sigma^* \in \{0,1\}$.

For the case of $\sigma^* = 1$, such an equilibrium requires the indifference condition $\Delta^{\widetilde{M}}(\varnothing) - \Delta(1) = 0$. Letting δ_R^η denote the probability, in the eyes of R, of S holding a signal η conditional on no disclosure, this is equivalent to:

$$\left|\delta_R^1(\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)) + (1-\delta_R^1)(\beta_S - \beta_R)\right| - \left|\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)\right| = 0 \Leftrightarrow \quad (21)$$

$$(1-\delta_R^1)\left(\overline{\beta} - \underline{\beta} - \frac{\overline{\beta}p}{\overline{\beta}p + (1-\overline{\beta})(1-p)} + \frac{\underline{\beta}p}{\underline{\beta}p + (1-\underline{\beta})(1-p)}\right) = 0 \Leftrightarrow \quad (22)$$

$$\overline{\beta} - \underline{\beta} - \frac{\overline{\beta}p}{\overline{\beta}p + (1-\overline{\beta})(1-p)} + \frac{\underline{\beta}p}{\underline{\beta}p + (1-\underline{\beta})(1-p)} = 0 \Leftrightarrow \quad (23)$$

$$\left\{\underline{\beta}, \frac{(1-\underline{\beta})(1-p)}{1-p+\underline{\beta}(2p-1)}\right\} = \overline{\beta} \Leftrightarrow \quad (24)$$

$$\left\{\overline{\beta}, \frac{(1-\overline{\beta})(1-p)}{1-p+\overline{\beta}(2p-1)}\right\} = \underline{\beta}. \quad (25)$$

Thus, given $\beta_S \neq \beta_R$, S is indifferent between sending 1 and no disclosure if and only if $\beta_S = \beta_S^*(\beta_R)$. It then holds true that $\Delta^{\widetilde{M}}(\varnothing) > \Delta(0)$ since

$$\min\{\left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right|, \left|\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)\right|\} < |\beta_S - \beta_R|$$

by Lemma II.A.

For the case of $\sigma^* = 0$, analogously, such an equilibrium requires the indifference condition $\Delta^{\widetilde{M}}(\varnothing) - \Delta^{\widetilde{M}}(0) = 0$. This is further equivalent to:

$$\left|\delta_R^0(\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)) + (1-\delta_R^0)(\beta_S - \beta_R)\right| - \left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right| = 0 \Leftrightarrow \quad (26)$$

$$(1-\delta_R^0)\left(\overline{\beta} - \underline{\beta} - \frac{\overline{\beta}(1-p)}{\overline{\beta}(1-p) + (1-\overline{\beta})p} + \frac{\underline{\beta}(1-p)}{\underline{\beta}(1-p) + (1-\underline{\beta})p}\right) = 0 \Leftrightarrow \quad (27)$$

$$\left(\overline{\beta} - \underline{\beta} - \frac{\overline{\beta}(1-p)}{\overline{\beta}(1-p) + (1-\overline{\beta})p} + \frac{\underline{\beta}(1-p)}{\underline{\beta}(1-p) + (1-\underline{\beta})p}\right) = 0 \Leftrightarrow \quad (28)$$

$$\left\{\underline{\beta}, \frac{p(1-\underline{\beta})}{\underline{\beta}+p(1-2\underline{\beta})}\right\} = \overline{\beta} \Leftrightarrow \tag{29}$$

$$\left\{\overline{\beta}, \frac{p(1-\overline{\beta})}{\overline{\beta}+p(1-2\underline{\beta})}\right\} = \underline{\beta}. \tag{30}$$

This similarly leads to $\beta_S = \beta_S^{**}(\beta_R)$. ■

Proof of Corollary 1.

Step 1. Point a) follows from the fact that $\beta_S^*(\beta_R) < 1-\beta_R < \beta_S^{**}(\beta_R)$. By Proposition 1, this means that for ε small enough, $\beta_S \in (1-\beta_R-\varepsilon, 1-\beta_R+\varepsilon)$ satisfies conditions such that the FD-equilibrium is the unique equilibrium.

Step 2. This proves Point b). Consider first $\beta_R < 1/2$ and β_S sufficiently close to β_R. Then, by Proposition 1, given that $\beta_S^*(\beta_R) < 1-\beta_R < \beta_S^{**}(\beta_R)$, the equilibrium features no full disclosure if and only if $\beta_R < \beta_S^*(\beta_R, p)$. In turn, for $\beta_R < 1/2$ we have

$$\begin{aligned} \beta_R &< \beta_S^*(\beta_R, p) \Leftrightarrow \\ p &< \frac{(1-\beta_R)^2}{(1-\beta_R)^2+(\beta_R)^2}. \end{aligned} \tag{31}$$

Hence, for any β_S sufficiently close to $\beta_R < 1/2$ under the above condition we obtain $\beta_S < \beta_S^*(\beta_R, p)$, in which case D0 is the unique equilibrium.

Consider $\beta_R > 1/2$ and β_S sufficiently close to β_R. Then, by Proposition 1, given that $\beta_S^*(\beta_R) < 1-\beta_R < \beta_S^{**}(\beta_R)$, the equilibrium features no full disclosure if and only if $\beta_R > \beta_S^{**}(\beta_R, p)$. In turn, for $\beta_R > 1/2$ we have

$$\begin{aligned} \beta_R &> \beta_S^{**}(\beta_R, p) \Leftrightarrow \\ p &< \frac{(\beta_R)^2}{(1-\beta_R)^2+(\beta_R)^2}. \end{aligned} \tag{32}$$

Hence, for any β_S sufficiently close to $\beta_R > 1/2$ under the above condition we obtain $\beta_S > \beta_S^{**}(\beta_R, p)$, in which case D1 is the unique equilibrium.

Finally, note that (31) and (32) combine into

$$\begin{aligned} \beta_R &\notin [\beta_S^*(\beta_R,p),\beta_S^{**}(\beta_R,p)] \Leftrightarrow \\ p &< \max\left\{\frac{(1-\beta_R)^2}{(1-\beta_R)^2+(\beta_R)^2},\frac{(\beta_R)^2}{(1-\beta_R)^2+(\beta_R)^2}\right\}. \end{aligned}$$

This together with Proposition 1 leads to the claim.

Step 3. Point c) follows due to $\beta_S^*(\beta_R,p)$ (resp. $\beta_S^{**}(\beta_R,p)$) being continuously decreasing (resp. increasing) in p and being equal to 0 (1) if $p=1$.

Step 4. This proves point d). Let $\beta_R<1/2$, i.e. R is biased towards 0.

By Proposition 1, a D1-equilibrium exists if and only if $\beta_S \geq \beta_S^{**}(\beta_R) > 1-\beta_R$. This implies that β_S is closer to 1 than β_R is close to 0, meaning that β_S is biased towards 1 and S is more confident than R.

By Proposition 1, a D0-equilibrium exists if and only if $\beta_S \leq \beta_S^*(\beta_R) < 1-\beta_R$. This in turn is compatible with two cases: Either β_R is the most confident prior or β_S is the most confident prior, in which case it also holds true that $\beta_S \leq \beta_R$. In both cases, note that that the most confident of the two priors is smaller than $\frac{1}{2}$, i.e. the more confident player is biased towards 0.

Let R be biased towards 1 ($\beta_R \geq 1/2$). The symmetric argument as given for the case of $\beta_R<1/2$ applies. ■

Appendix III: Propositions 3 and 4

Proof of Proposition 3

Step 1. Consider the case $\beta_S>\beta_R$ in D0-equilibrium. From S's ex ante perspective, the expected ex post perceived disagreement is

$$\begin{aligned} E_S[\Delta^{D0}] &= (\Pr[\sigma=1\,|\beta_S]+\Pr[\sigma=\varnothing\,|\beta_S])(E_R^{D0}[\widetilde{\beta}_S|\varnothing]-\widetilde{\beta}_R^{D0}(\varnothing)) \\ &\quad +\Pr[\sigma=0\,|\beta_S](\widetilde{\beta}_S(0)-\widetilde{\beta}_R(0)). \end{aligned}$$

At the same time, under full disclosure

$$\begin{aligned} E_S[\Delta^{FD}] &= \Pr[\sigma = 1\,|\beta_S](\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)) + \Pr[\sigma = 0\,|\beta_S](\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)) \\ &\quad + \Pr[\sigma = \varnothing\,|\beta_S](\beta_S - \beta_R). \end{aligned}$$

Using the expressions obtained in Appendix I, it follows that:

$$\begin{aligned} & E_S[\Delta^{D0}] - E_S[\Delta^{FD}] \\ = \; & \Pr[\sigma = 1\,|\beta_S](E_R^{D0}[\widetilde{\beta}_S|\varnothing] - \widetilde{\beta}_R^{D0}(\varnothing) - (\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1))) \\ & + \Pr[\sigma = \varnothing\,|\beta_S](E_R^{D0}[\widetilde{\beta}_S|\varnothing] - \widetilde{\beta}_R^{D0}(\varnothing) \\ & -(\beta_S - \beta_R)) \\ = \; & \varphi(\beta_S p + (1-\beta_S)(1-p)) \\ & \times \left(\begin{array}{c} \left(\frac{\varphi(\beta_R p + (1-\beta_R)(1-p))}{\varphi(\beta_R p + (1-\beta_R)(1-p)) + (1-\varphi)} - 1\right)\left(\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)\right) \\ + \left(\frac{(1-\varphi)}{\beta_R\varphi p + (1-\beta_R)\varphi(1-p) + (1-\varphi)}\right)(\beta_S - \beta_R) \end{array} \right) \\ & + (1-\varphi) \left(\begin{array}{c} \left(\frac{\varphi(\beta_R p + (1-\beta_R)(1-p))}{\varphi(\beta_R p + (1-\beta_R)(1-p)) + (1-\varphi)}\right)\left(\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)\right) \\ + \left(\frac{(1-\varphi)}{\beta_R\varphi p + (1-\beta_R)\varphi(1-p) + (1-\varphi)} - 1\right)(\beta_S - \beta_R) \end{array} \right) \\ = \; & \Phi_1\Phi_2 \end{aligned}$$

where

$$\begin{aligned} \Phi_1 &= \frac{(\beta_S - \beta_R)^2(1-2p)^2(1-\varphi)\varphi}{(\beta_R p + (1-\beta_R)(1-p))(\beta_S p + (1-\beta_S)(1-p))(1 - p\varphi + \beta_R\varphi(2p-1))} > 0, \\ \Phi_2 &= (\beta_R + \beta_S - 1)(1-p) + \beta_R\beta_S(2p-1). \end{aligned}$$

Note that Φ_2 is an increasing function of β_S. At the same time, by Proposition 1, it must be true that $\beta_S < \beta_S^*$ if the D0-equilibrium is the unique equilibrium. Consequently,

$$\begin{aligned} \Phi_2(\beta_S) &< \Phi_2(\beta_S^*) = \left(\beta_R + \frac{(1-\beta_R)(1-p)}{1-p+\beta_R(2p-1)} - 1\right)(1-p) \\ &\quad + \beta_R\frac{(1-\beta_R)(1-p)}{1-p+\beta_R(2p-1)}(2p-1) \\ &= 0. \end{aligned}$$

Hence, $\Phi_1\Phi_2 < 0$ so that

$$E_S[\Delta^{D0}] - E_S[\Delta^{FD}] < 0,$$

i.e. the sender would ex ante prefer D0 over FD.

Step 2. Consider the case $\beta_S > \beta_R$ in D1-equilibrium. From S's perspective, the ex ante expected ex post perceived disagreement is

$$\begin{aligned} E_S[\Delta^{D1}] &= (\Pr[\sigma = 0\,|\beta_S] + \Pr[\sigma = \varnothing\,|\beta_S])(E_R^{D1}[\widetilde{\beta}_S|\varnothing] - \widetilde{\beta}_R^{D1}(\varnothing)) \\ &\quad + \Pr[\sigma = 1\,|\beta_S](\widetilde{\beta}_S(1) - \widetilde{\beta}_R(1)) \end{aligned}$$

It follows that

$$\begin{aligned} & E_S[\Delta^{D1}] - E_S[\Delta^{FD}] \\ = \ & \varphi(\beta_S(1-p) + (1-\beta_S)p) \\ & \times \left(\begin{array}{c} \left(\frac{\varphi(\beta_R(1-p)+(1-\beta_R)p)}{\varphi(\beta_R(1-p)+(1-\beta_R)p)+(1-\varphi)} - 1\right)\left(\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right) \\ + \left(\frac{(1-\varphi)}{\varphi(\beta_R(1-p)+(1-\beta_R)p)+(1-\varphi)}\right)(\beta_S - \beta_R) \end{array} \right) \\ & +(1-\varphi) \left(\begin{array}{c} \left(\frac{\varphi(\beta_R(1-p)+(1-\beta_R)p)}{\varphi(\beta_R(1-p)+(1-\beta_R)p)+(1-\varphi)}\right)\left(\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right) \\ + \left(\frac{(1-\varphi)}{\varphi(\beta_R(1-p)+(1-\beta_R)p)+(1-\varphi)} - 1\right)(\beta_S - \beta_R) \end{array} \right) \\ = \ & \Phi_3\Phi_4, \end{aligned}$$

where

$$\begin{aligned} \Phi_3 &= -\frac{(\beta_S - \beta_R)^2(1-2p)^2(1-\varphi)\varphi}{(\beta_R(1-p) + (1-\beta_R)p)(\beta_S(1-p) + (1-\beta_S)p)} \\ &\quad \times \frac{1}{1 - \varphi((1-\beta_R)(1-p) + \beta_R p)} \\ &< 0, \\ \Phi_4 &= p(1-\beta_R) - \beta_S(p(1-\beta_R) + \beta_R(1-p)). \end{aligned}$$

Function Φ_4 is decreasing in β_S. At the same time, by Proposition 1 it must be true that $\beta_S > \beta_S^{**}$ if the D1-equilibrium is the unique equilibrium. Consequently,

$$\Phi_4(\beta_S) < \Phi_4(\beta_S^{**}) = p(1-\beta_R) - \frac{p(1-\beta_R)}{\beta_R + p(1-2\beta_R)}(p(1-\beta_R) + \beta_R(1-p)) = 0.$$

Hence, $\Phi_3\Phi_4 > 0$, i.e.

$$E_S[\Delta^{D1}] - E_S[\Delta^{FD}] > 0,$$

i.e. the sender would ex ante prefer FD over D1.

Step 3. Consider the case $\beta_S < \beta_R$. Then, it can be shown that

$$\begin{aligned} E_S[\Delta^{D0}] - E_S[\Delta^{FD}] &= -\Phi_1\Phi_2 > 0, \\ E_S[\Delta^{D1}] - E_S[\Delta^{FD}] &= -\Phi_3\Phi_4 < 0. \end{aligned}$$

Thus, the sender would ex ante prefer FD over D0 and D1 over FD whenever D0 and D1 are the unique equilibria, respectively. ■

Proof of Proposition 4

Step 1. In Steps 1-4 below, we consider the case that $\beta_S > \beta_R$. Define as $\widetilde{\Theta}(\text{Partial},\widehat{\beta})$ and $\widetilde{\Theta}(\text{Full},\widehat{\beta})$ the expected actual disagreement under partial and full disclosure respectively, from the perspective of a third party endowed with prior $\widehat{\beta}$. Denote further by $\widetilde{\beta}_i(\iota,\text{Partial})$ and $\widetilde{\beta}_i(\iota,\text{Full})$ the posterior of player i conditional on obtained information ι under partial and full disclosure respectively. We have:

$$\begin{aligned} \widetilde{\Theta}(\text{Partial},\widehat{\beta}) &= E_{\widehat{\beta}}\left[\left|\widetilde{\beta}_S(\sigma,\text{Partial}) - \widetilde{\beta}_R(d,\text{Partial})\right|\right] \\ &\geq E_{\widehat{\beta}}\left[\widetilde{\beta}_S(\sigma,\text{Partial}) - \widetilde{\beta}_R(d,\text{Partial})\right] \\ &= E_{\widehat{\beta}}[\widetilde{\beta}_S(\sigma,\text{Partial})] - E_{\widehat{\beta}}[\widetilde{\beta}_R(d,\text{Partial})] \\ &= E_{\widehat{\beta}}\left[\widetilde{\beta}_S(\sigma,\text{Full})\right] - E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d,\text{Partial})\right]. \end{aligned} \tag{33}$$

In the above, the equality $E_{\widehat{\beta}}[\widetilde{\beta}_S(\sigma,\text{Partial})] = E_{\widehat{\beta}}\left[\widetilde{\beta}_S(\sigma,\text{Full})\right]$ follows from the fact that S's expected posterior is independent of the disclosure rule. Note on the other hand that

$$\begin{aligned} \widetilde{\Theta}(\text{Full},\widehat{\beta}) &= E_{\widehat{\beta}}\left[\left|\widetilde{\beta}_S(\sigma,\text{Full}) - \widetilde{\beta}_R(d,\text{Full})\right|\right] \\ &= E_{\widehat{\beta}}\left[\widetilde{\beta}_S(\sigma,\text{Full})\right] - E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d,\text{Full})\right]. \end{aligned} \tag{34}$$

To see this, note that under FD, it always holds true that $d = \sigma$. Recall also that $\widetilde{\beta}_S(\sigma) > \widetilde{\beta}_R(\sigma)$ for any σ given $\beta_S > \beta_R$.

It follows from the above analysis that

$$\widetilde{\Theta}(\text{Partial},\widehat{\beta}) - \widetilde{\Theta}(\text{Full},\widehat{\beta}) \geq E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d, \text{Full})\right] - E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d, \text{Partial})\right]. \tag{35}$$

Step 2. We now show that $E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d, \text{Full})\right] - E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d, \text{Partial})\right] > 0$ if and only if $\widehat{\beta} > \beta_R$. Here we follow the steps of the analysis presented in Kartik et al. (2015). One can verify that

$$\widetilde{\beta}_R(d) = \frac{\widetilde{\beta}(d)\frac{\beta_R}{\beta}}{\widetilde{\beta}(d)\frac{\beta_R}{\beta} + (1 - \widetilde{\beta}(d))\frac{1-\beta_R}{1-\beta}},$$

where $\widetilde{\beta}(d)$ denotes the hypothetical posterior belief of R if she had a prior $\widehat{\beta}$ and observed d. One can verify that the above function is concave in $\widetilde{\beta}(d)$ if $\beta < \beta_R$ and convex if the opposite inequality holds. Blackwell (1953) has shown that a garbling increases (resp. reduces) an individual's expectation of any concave (resp. convex) function of his posterior. Then, since partial disclosure is a garbling of full disclosure,[31] we obtain that

$$E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d, \text{Partial})\right] < (>) E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d, \text{Full})\right] \text{ if } \widehat{\beta} > (<)\beta_R \tag{36}$$

given that R's posterior is a convex (concave) function of $\widetilde{\widehat{\beta}}(\sigma)$ if $\widehat{\beta} > (<)\beta_R$.

Step 3. (35) and (36) together imply

$$\widetilde{\Theta}(\text{Partial},\widehat{\beta}) - \widetilde{\Theta}(\text{Full},\widehat{\beta}) > 0 \text{ if } \widehat{\beta} > \beta_R.$$

Thus, the third party would prefer full disclosure over partial disclosure whenever $\widehat{\beta} > \beta_R$, i.e. whenever $\beta_R < \widehat{\beta} < \beta_S$ or $\widehat{\beta} \geq \beta_S > \beta_R$.

[31] See Kartik et al. (2015) for a formal definition of garbling.

Step 4. Consider $\widehat{\beta} < \beta_R < \beta_S$. If β_S is sufficiently close to 1, then we have:

$$\begin{aligned}\widetilde{\Theta}(\text{Partial},\widehat{\beta}) &= E_{\widehat{\beta}}\left[\left|\widetilde{\beta}_S(\sigma,\text{Partial}) - \widetilde{\beta}_R(d,\text{Partial})\right|\right] \\ &= E_{\widehat{\beta}}\left[\widetilde{\beta}_S(\sigma,\text{Partial}) - \widetilde{\beta}_R(d,\text{Partial})\right] \\ &= E_{\widehat{\beta}}[\widetilde{\beta}_S(\sigma,\text{Partial})] - E_{\widehat{\beta}}[\widetilde{\beta}_R(d,\text{Partial})] \\ &= E_{\widehat{\beta}}\left[\widetilde{\beta}_S(\sigma,\text{Full})\right] - E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d,\text{Partial})\right].\end{aligned}$$

Note in the above that we have equalities at all stages in contrast to (33). This together with (34) and (36) implies

$$\widetilde{\Theta}(\text{Partial},\widehat{\beta}) - \widetilde{\Theta}(\text{Full},\widehat{\beta}) = E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d,\text{Full})\right] - E_{\widehat{\beta}}\left[\widetilde{\beta}_R(d,\text{Partial})\right] < 0.$$

Hence, in this case the third party would prefer partial disclosure over full disclosure in terms of minimizing expected actual disagreement.

Step 5. The proof for the remaining case of $\beta_S < \beta_R$ is conceptually identical to what has been presented, and is hence omitted. We obtain the following counterparts of the statements proven above:

$$\begin{aligned}\widetilde{\Theta}(\text{Partial},\widehat{\beta}) - \widetilde{\Theta}(\text{Full},\widehat{\beta}) &> 0 \text{ if } \widehat{\beta} < \beta_R, \\ \widetilde{\Theta}(\text{Partial},\widehat{\beta}) - \widetilde{\Theta}(\text{Full},\widehat{\beta}) &< 0 \text{ if } \beta_S < \beta_R < \widehat{\beta} \text{ and } \beta_S \text{ is close to 0.}\end{aligned}$$

■

Appendix IV: Proposition 5

Proof of Proposition 5.a)

Step 1. Consider a putative FD-equilibrium. Let $G_S(G_R)$ denote the (symmetric) cumulative distribution function of S's (R's) prior belief. Then, if the sender discloses 0-signal, the receiver with the prior β_R believes that the disagreement is

$$\Delta(0) = \int_{\beta_S=0}^{1} \left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right| dG_S(\beta_S).$$

In turn, the sender expects that the receiver's perceived disagreement is

$$E_S[\Delta(0)] = \int_{\beta_R=0}^{1} \int_{\beta_S=0}^{1} \left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right| dG_S(\beta_S) dG_R(\beta_R).$$

If the sender does not disclose, the expected perceived disagreement is

$$E_S[\Delta^{FD}(\varnothing)] = \int_{\beta_R=0}^{1} \int_{\beta_S=0}^{1} |\beta_S - \beta_R| dG_S(\beta_S) dG_R(\beta_R).$$

In FD-equilibrium we must have $E_S[\Delta(0)] - E_S[\Delta^{FD}(\varnothing)] < 0$. We have

$$\begin{aligned} & E_S[\Delta(0)] - E_S[\Delta^{FD}(\varnothing)] \\ = & \int_{\beta_R=0}^{1} \int_{\beta_S=0}^{1} \left(\left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right| - |\beta_S - \beta_R|\right) dG_S(\beta_S) dG_R(\beta_R). \end{aligned}$$

Denote $\widetilde{\beta}(\sigma, \beta)$ the posterior belief given obtained/disclosed signal σ and prior belief β. Besides, denote $\kappa(\beta_i, \beta_j) = \left|\widetilde{\beta}(0, \beta_i) - \widetilde{\beta}(0, \beta_j)\right| - |\beta_i - \beta_j|$. Then,

$$\begin{aligned} & \int_{\beta_R=0}^{1} \int_{\beta_S=0}^{1} \left(\left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right| - |\beta_S - \beta_R|\right) dG_S(\beta_S) dG_R(\beta_R) \\ = & \int_{\beta_R=0}^{1} \int_{\beta_S=0}^{1} \kappa(\beta_S, \beta_R) dG_S(\beta_S) dG_R(\beta_R) \\ = & \int_{\beta_R=0}^{0.5} \int_{\beta_S=0}^{1} \kappa(\beta_S, \beta_R) dG_S(\beta_S) dG_R(\beta_R) \\ & + \int_{\beta_R=0}^{0.5} \int_{\beta_S=0}^{1} \kappa(\beta_S, 1 - \beta_R) dG_S(\beta_S) dG_R(1 - \beta_R) \\ = & \int_{\beta_R=0}^{0.5} \int_{\beta_S=0}^{1} \kappa(\beta_S, \beta_R) dG_S(\beta_S) dG_R(\beta_R) \\ & + \int_{\beta_R=0}^{0.5} \int_{\beta_S=0}^{1} \kappa(\beta_S, 1 - \beta_R) dG_S(\beta_S) dG_R(\beta_R) \\ = & \int_{\beta_R=0}^{0.5} \int_{\beta_S=0}^{1} \left(\kappa(\beta_S, \beta_R) + \kappa(\beta_S, 1 - \beta_R)\right) dG_S(\beta_S) dG_R(\beta_R), \end{aligned}$$

where the third equality follows due to symmetry of G. Next, denote $\lambda(\beta_S, \beta_R) = \kappa(\beta_S, \beta_R) + \kappa(\beta_S, 1-\beta_R)$. Then, similarly,

$$\begin{aligned}
& \int_{\beta_R=0}^{0.5} \int_{\beta_S=0}^{1} \left(\kappa(\beta_S, \beta_R) + \kappa(\beta_S, 1-\beta_R)\right) dG_S(\beta_S) dG_R(\beta_R) \\
= & \int_{\beta_R=0}^{0.5} \int_{\beta_S=0}^{1} \lambda(\beta_S, \beta_R) dG_S(\beta_S) dG_R(\beta_R) \\
= & \int_{\beta_R=0}^{0.5} \begin{pmatrix} \int_{\beta_S=0}^{0.5} \lambda(\beta_S, \beta_R) dG_S(\beta_S) \\ + \int_{\beta_S=0}^{0.5} \lambda(1-\beta_S, \beta_R) dG_S(1-\beta_S) \end{pmatrix} dG_R(\beta_R) \\
= & \int_{\beta_R=0}^{0.5} \int_{\beta_S=0}^{0.5} (\lambda(\beta_S, \beta_R) + \lambda(1-\beta_S, \beta_R)) dG_S(\beta_S) dG_R(\beta_R).
\end{aligned}$$

Let us now show that $\lambda(\beta_S, \beta_R) + \lambda(1-\beta_S, \beta_R) < 0$ for any $\beta_S < 0.5$ and $\beta_R < 0.5$ in which case the whole integral on the right-hand side is negative. Denote as before $\overline{\beta} = \max\{\beta_S, \beta_R\}$ and $\underline{\beta} = \min\{\beta_S, \beta_R\}$. Then, (noting that $1-\underline{\beta} > 1-\overline{\beta} > \overline{\beta} > \underline{\beta}$ due to both $\overline{\beta} < 0.5$ and $\underline{\beta} < 0.5$)

$$\begin{aligned}
& \lambda(\beta_S, \beta_R) + \lambda(1-\beta_S, \beta_R) \\
= & \kappa(\beta_S, \beta_R) + \kappa(\beta_S, 1-\beta_R) + \kappa(1-\beta_S, \beta_R) + \kappa(1-\beta_S, 1-\beta_R) \\
= & \left(\widetilde{\beta}(0, \overline{\beta}) - \widetilde{\beta}(0, \underline{\beta})\right) - (\overline{\beta} - \underline{\beta}) \\
& + \left(\widetilde{\beta}(0, 1-\overline{\beta}) - \widetilde{\beta}(0, \underline{\beta})\right) - (1 - \overline{\beta} - \underline{\beta}) \\
& + \left(\widetilde{\beta}(0, 1-\underline{\beta}) - \widetilde{\beta}(0, \overline{\beta})\right) - (1 - \underline{\beta} - \overline{\beta}) \\
& + \left(\widetilde{\beta}(0, 1-\underline{\beta}) - \widetilde{\beta}(0, 1-\overline{\beta})\right) - (1 - \underline{\beta} - (1-\overline{\beta})) \\
= & 2(\widetilde{\beta}(0, 1-\underline{\beta}) - \widetilde{\beta}(0, \underline{\beta}) + 2\underline{\beta} - 1) \\
= & 2\left(\frac{(1-\underline{\beta})(1-p)}{(1-\underline{\beta})(1-p) + \underline{\beta}p} - \frac{\underline{\beta}(1-p)}{\underline{\beta}(1-p) + (1-\underline{\beta})p} + 2\underline{\beta} - 1\right) \\
= & -\frac{2(1-2p)^2(1-\underline{\beta})(1-2\underline{\beta})\underline{\beta}}{(1-p+\underline{\beta}(2p-1))(\underline{\beta} + p(1-2\underline{\beta}))} < 0,
\end{aligned}$$

where the inequality follows due to $\underline{\beta} < 0.5$.

Step 2. By symmetry considerations, the same property holds for 1-signals, i.e. $E_S E_R[\Delta(1)] - E_S E_R[\Delta^{FD}(\varnothing)] < 0$. Formally, the proof proceeds analogously redefining $\kappa(\beta_i,\beta_j)=\left|\widetilde{\beta}(1,\beta_i)-\widetilde{\beta}(1,\beta_j)\right|-|\beta_i-\beta_j|$. ■

Proof of Proposition 5.b)

In what follows, we assume without loss of generality that MLRP is satisfied as

$$\frac{\partial}{\partial x}\frac{g_S(x)}{g_R(x)}>0. \tag{37}$$

Step 1. Denote the difference in disagreement under disclosure and no disclosure in a putative FD-equilibrium as

$$\begin{aligned}\kappa_0(\beta_S,\beta_R) &= |\beta_S-\beta_R|-\left|\widetilde{\beta}(0,\beta_S)-\widetilde{\beta}(0,\beta_R)\right|,\\ \kappa_1(\beta_S,\beta_R) &= |\beta_S-\beta_R|-\left|\widetilde{\beta}(1,\beta_S)-\widetilde{\beta}(1,\beta_R)\right|.\end{aligned}$$

In FD, we have

$$\begin{aligned}\int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1}\kappa_0(\beta_S,\beta_R)g_S(\beta_S)g_R(\beta_R)d\beta_S d\beta_R &\geq 0,\\ \int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1}\kappa_1(\beta_S,\beta_R)g_S(\beta_S)g_R(\beta_R)d\beta_S d\beta_R &\geq 0.\end{aligned}$$

Since the joint distribution of priors is completely symmetric with respect to either boundary (0 or 1), the effect of 0-disclosure on the expected disagreement should be equivalent to the effect of 1-disclosure, i.e.

$$\begin{aligned}&\int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1}\kappa_0(\beta_S,\beta_R)g_S(\beta_S)g_R(\beta_R)d\beta_S d\beta_R\\ =&\int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1}\kappa_1(\beta_S,\beta_R)g_S(\beta_S)g_R(\beta_R)d\beta_S d\beta_R.\end{aligned}$$

This implies that for $i=0,1$

$$\begin{aligned}\int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1}\kappa_i(\beta_S,\beta_R)g_S(\beta_S)g_R(\beta_R)d\beta_S d\beta_R &\geq 0\Leftrightarrow\\ \int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1}\eta(\beta_S,\beta_R)g_S(\beta_S)g_R(\beta_R)d\beta_S d\beta_R &\geq 0.\end{aligned}$$

where $\eta(\beta_S, \beta_R) = \kappa_0(\beta_S, \beta_R) + \kappa_1(\beta_S, \beta_R)$.

Step 2. We have

$$\begin{aligned}
&\int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1} \eta(\beta_S,\beta_R) g_S(\beta_S) g_R(\beta_R) d\beta_S d\beta_R \\
= &\int_{\beta_R=0}^{0.5}\int_{\beta_S=0}^{1} \eta(\beta_S,\beta_R) g_S(\beta_S) g_R(\beta_R) d\beta_S d\beta_R \\
&+\int_{\beta_R=0}^{0.5}\int_{\beta_S=0}^{1} \eta(\beta_S,1-\beta_R) g_S(\beta_S) g_R(1-\beta_R) d\beta_S d\beta_R \\
= &\int_{\beta_R=0}^{0.5}\int_{\beta_S=0}^{0.5} \eta(\beta_S,\beta_R) g_S(\beta_S) g_R(\beta_R) d\beta_S d\beta_R \\
&+\int_{\beta_R=0}^{0.5}\int_{\beta_S=0}^{0.5} \eta(1-\beta_S,\beta_R) g_S(1-\beta_S) g_R(\beta_R) d\beta_S d\beta_R \\
&+\int_{\beta_R=0}^{0.5}\int_{\beta_S=0}^{0.5} \eta(\beta_S,1-\beta_R) g_S(\beta_S) g_R(1-\beta_R) d\beta_S d\beta_R \\
&+\int_{\beta_R=0}^{0.5}\int_{\beta_S=0}^{0.5} \eta(1-\beta_S,1-\beta_R) g_S(1-\beta_S) g_R(1-\beta_R) d\beta_S d\beta_R \\
= &\int_{\beta_R=0}^{0.5}\int_{\beta_S=0}^{0.5} \varsigma(\beta_S,\beta_R) d\beta_S d\beta_R,
\end{aligned}$$

where

$$\begin{aligned}
\varsigma(\beta_S,\beta_R) = \; &\eta(\beta_S,\beta_R) g_S(\beta_S) g_R(\beta_R) + \eta(1-\beta_S,\beta_R) g_S(1-\beta_S) g_R(\beta_R) \\
&+\eta(\beta_S,1-\beta_R) g_S(\beta_S) g_R(1-\beta_R) + \eta(1-\beta_S,1-\beta_R) g_S(1-\beta_S) g_R(1-\beta_R).
\end{aligned}$$

Hence, given Step 1, for the main claim it is sufficient to show that $\varsigma(\beta_S, \beta_R) \geq 0$ for any $\{\beta_S, \beta_R\} \in [0, 0.5]^2$.

Step 3. Let us show that $\varsigma(\beta_S, \beta_R)$ is increasing in p for $p \in (1/2, 1)$ and $\{\beta_S, \beta_R\} \in [0, 0.5]^2$. To simplify the notation, let us denote $g_S(\beta_S) \equiv g_{S1}$, $g_R(\beta_R) \equiv g_{R1}$, $g_S(1-\beta_S) \equiv g_{S2}$, $g_R(1-\beta_R) \equiv g_{R2}$.

Consider first $0.5 \geq \beta_R > \beta_S$. Substituting all expressions into $\varsigma(\beta_S, \beta_R)$ and simplifying, we obtain

$$
\begin{aligned}
\varsigma(\beta_S, \beta_R) &= \tau_1(\beta_R + \beta_S - 1)(g_{R2}g_{S1} + g_{R1}g_{S2}) \\
&\quad +\tau_2(\beta_R - \beta_S)(g_{R1}g_{S1} + g_{R2}g_{S2}),
\end{aligned}
$$

where

$$
\begin{aligned}
\tau_1 &= -2 - \frac{(1-p)p}{(\beta_R + p - 2\beta_R p)(\beta_S + p - 2\beta_S p - 1)} \\
&\quad - \frac{(1-p)p}{(\beta_R + p - 2\beta_R p - 1)(\beta_S + p - 2\beta_S p)}, \\
\tau_2 &= 2 - \frac{(1-p)p}{(\beta_R + p - 2\beta_R p - 1)(\beta_S + p - 2\beta_S p - 1)} \\
&\quad - \frac{(1-p)p}{(\beta_R + p - 2\beta_R p)(\beta_S + p - 2\beta_S p)}.
\end{aligned}
$$

Taking the derivative of $\varsigma(\beta_S, \beta_R)$ with respect to p and simplifying we obtain

$$
\begin{aligned}
\frac{\partial \varsigma(\beta_S, \beta_R)}{\partial p} &= T_1 + T_2, \\
T_1 &= (1-\beta_R)\beta_R \frac{(2p-1)(1-2\beta_R)}{(\beta_R + p - 2\beta_R p - 1)^2(\beta_R + p - 2\beta_R p)^2} \\
&\quad \times (g_{R2} - g_{R1})(g_{S1} - g_{S2}), \\
T_2 &= (1-\beta_S)\beta_S \frac{(2p-1)(1-2\beta_S)}{(\beta_S + p - 2\beta_S p - 1)^2(\beta_S + p - 2\beta_S p)^2} \\
&\quad \times (g_{R2} + g_{R1})(g_{S1} + g_{S2}).
\end{aligned}
$$

Consider now the case $\beta_R < \beta_S \leq 0.5$. Substituting all expressions into $\varsigma(\beta_S, \beta_R)$ and simplifying, we obtain in this case

$$
\begin{aligned}
\varsigma(\beta_S, \beta_R) &= \tau_1(\beta_R + \beta_S - 1)(g_{R2}g_{S1} + g_{R1}g_{S2}) \\
&\quad +\tau_2(\beta_S - \beta_R)(g_{R1}g_{S1} + g_{R2}g_{S2}),
\end{aligned}
$$

Taking the derivative of $\varsigma(\beta_S, \beta_R)$ with respect to p and simplifying we obtain

$$
\frac{\partial \varsigma(\beta_S, \beta_R)}{\partial p} = \widehat{T}_1 + \widehat{T}_2,
$$

where

$$
\begin{aligned}
\widehat{T}_1 &= (1-\beta_R)\beta_R \frac{(2p-1)(1-2\beta_R)}{(\beta_R+p-2\beta_R p-1)^2(\beta_R+p-2\beta_R p)^2} \\
&\times (g_{R2}+g_{R1})(g_{S1}+g_{S2}), \\
\widehat{T}_2 &= (1-\beta_R)\beta_R \frac{(2p-1)(1-2\beta_S)}{(\beta_S+p-2\beta_S p-1)^2(\beta_S+p-2\beta_S p)^2} \\
&\times (g_{R2}-g_{R1})(g_{S1}-g_{S2}).
\end{aligned}
$$

Recall that $\{\beta_S, \beta_R\} \in [0, 0.5]^2$ by assumption. Hence, to show that $\frac{\partial \varsigma(\beta_S,\beta_R)}{\partial p} \geq 0$ in either case we need to show that

$$(g_{R2}-g_{R1})(g_{S1}-g_{S2}) > 0.$$

This is done in the next step.

Step 4. By initial assumption, we have that for any x

$$g_R(x) = g_S(1-x).$$

In particular, this implies

$$\frac{g_R(0.5)}{g_S(0.5)} = 1.$$

Note that then the MLRP in (37) implies that for any $\beta_R < 0.5$ and $\beta_S < 0.5$

$$
\begin{aligned}
\frac{g_S(\beta_S)}{g_R(\beta_S)} &< \frac{g_S(0.5)}{g_R(0.5)} < \frac{g_S(1-\beta_R)}{g_R(1-\beta_R)} \Leftrightarrow \\
\frac{g_S(\beta_S)}{g_R(\beta_S)} &< 1 < \frac{g_S(1-\beta_R)}{g_R(1-\beta_R)}.
\end{aligned} \tag{38}
$$

Since by initial assumption $g_R(x) = g_S(1-x)$, (38) is equivalent to

$$\frac{g_S(\beta_S)}{g_S(1-\beta_S)} < 1 < \frac{g_R(\beta_R)}{g_R(1-\beta_R)}.$$

In terms of our previous notation, this is equivalent to

$$
\begin{aligned}
g_{S1} &< g_{S2}, \\
g_{R1} &> g_{R2}.
\end{aligned}
$$

Finally, this leads to

$$(g_{R2} - g_{R1})(g_{S1} - g_{S2}) > 0. \tag{39}$$

Step 5. Applying (39) to the expressions for $\frac{\partial \varsigma(\beta_S, \beta_R)}{\partial p}$ from Step 3, we obtain

$$\frac{\partial \varsigma(\beta_S, \beta_R)}{\partial p} \geq 0.$$

At the same time, it is easy to verify that $\varsigma(\beta_S, \beta_R) = 0$ for $p = 1/2$. Consequently, $\varsigma(\beta_S, \beta_R) \geq 0$ for any $p > 1/2$. Then, by Step 2 this results in

$$\int_{\beta_R=0}^{1} \int_{\beta_S=0}^{1} \eta(\beta_S, \beta_R) g_S(\beta_S) g_R(\beta_R) d\beta_S d\beta_R \geq 0.$$

By Step 1, this implies that the incentive constraints for full disclosure are satisfied. ■

Proof of Proposition 5.c)

Step 1. Let us show that for sufficiently high x, it holds that $\beta_S > \beta_S^{**}(\beta_R) = \frac{p(1-\beta_R)}{\beta_R + p(1-2\beta_R)}$ for any $\{\beta_S, \beta_R\} \in [x, 1]^2$. Indeed, it is easy to verify that $x > \beta_S^{**}(x)$ if and only if $x > \frac{p}{p+\sqrt{p(1-p)}}$. Thus, we have that for $x > \frac{p}{p+\sqrt{p(1-p)}}$ and any $\{\beta_S, \beta_R\} \in [x, 1]^2$ it holds

$$\beta_S \geq x > \beta^{**}(x) \geq \beta^{**}(\beta_R),$$

where the last inequality is due to $\beta_S^{**}(x)$ decreasing in x. Hence, $\beta_S > \beta_S^{**}(\beta_R)$ for any $\{\beta_S, \beta_R\} \in [x, 1]^2$.

Analogously, one can show that for any sufficiently small y (in particular, for any $y < \frac{p+\sqrt{p(1-p)}-1}{2p-1}$), it holds $\beta_S < \beta_S^{*}(\beta_R)$ for any $\{\beta_S, \beta_R\} \in [0, y]^2$.

Step 2. Let us show that if the common distribution of priors g is shifted to the right, then D1-equilibrium always exists. The incentive constraints for D1 are (see

Step 1 in the proof of Proposition 5.a)

$$
\begin{aligned}
\int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1} \widetilde{\kappa}_0(\beta_S,\beta_R)g(\beta_S)g(\beta_R)d\beta_S d\beta_R &\leq 0, & (40)\\
\int_{\beta_R=0}^{1}\int_{\beta_S=0}^{1} \widetilde{\kappa}_1(\beta_S,\beta_R)g(\beta_S)g(\beta_R)d\beta_S d\beta_R &\geq 0, & (41)
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{\kappa}_0(\beta_S,\beta_R) &= \Delta^{D1}(\varnothing;\beta_S,\beta_R) - \Delta(0;\beta_S,\beta_R),\\
\widetilde{\kappa}_1(\beta_S,\beta_R) &= \Delta^{D1}(\varnothing;\beta_S,\beta_R) - \Delta(1;\beta_S,\beta_R).
\end{aligned}
$$

At the same time, for any constellation $\{\beta_S,\beta_R\} \in [x,1]^2$ and x sufficiently high we have $\beta_S > \beta_S^{**}(\beta_R)$ by Step 1, which then implies by Proposition 1

$$
\begin{aligned}
\widetilde{\kappa}_0(\beta_S,\beta_R) &\leq 0,\\
\widetilde{\kappa}_1(\beta_S,\beta_R) &\geq 0.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
\int_{\beta_R=x}^{1}\int_{\beta_S=x}^{1} \kappa_0(\beta_S,\beta_R)g_S(\beta_S)g_R(\beta_R)d\beta_S d\beta_R &\leq 0, & (42)\\
\int_{\beta_R=x}^{1}\int_{\beta_S=x}^{1} \kappa_1(\beta_S,\beta_R)g_S(\beta_S)g_R(\beta_R)d\beta_S d\beta_R &\geq 0. & (43)
\end{aligned}
$$

Finally, (42) and (43) result in (40) and (41) as far as g is sufficiently skewed to the right.

Step 3. The non-existence of other pure strategy equilibria (besides D1) if g is sufficiently shifted to the right follows by the analogous argument. In particular, by Step 1 and Proposition 1 for any given constellation $\{\beta_S,\beta_R\} \in [x,1]^2$ with x sufficiently high the S's incentive constraints for other equilibria (D0 and FD) are not satisfied. Consequently, they are still not satisfied once we integrate them over all possible constellations $\{\beta_S,\beta_R\} \in [x,1]^2$ like in Step 2. If the probability mass set on $\{\beta_S,\beta_R\} \notin [x,1]^2$ gets sufficiently small, the same applies to the integration over all possible constellations $\{\beta_S, \beta_R\} \in [0,1]^2$.

Step 4. Consider the case when the distribution g is sufficiently skewed to the left, i.e. to values $[0,y]^2$. As before, Step 1 implies that for any given $\{\beta_S, \beta_R\} \in [0,y]^2$ we have $\beta_S < \beta_S^*(\beta_R)$, i.e. the S's incentive constraints for D0 are satisfied, while for D1 and FD they are not satisfied. Consequently, the same holds once we integrate them over all possible priors constellations in $[0,y]^2$, and hence in $[0,1]^2$ (under sufficiently skewed distribution).

Proof of Proposition 5.d)

Suppose that S's prior β_S is commonly known. That of R is drawn from a symmetric distribution G over $[0,1]$. Then, by the same steps as in the proof of Proposition 5.a we obtain

$$\begin{aligned} E_S[\Delta(0)] - E_S[\Delta^{FD}(\varnothing)] &= \int_{\beta_R=0}^{1} \left(\left|\widetilde{\beta}_S(0) - \widetilde{\beta}_R(0)\right| - |\beta_S - \beta_R| \right) dG_R(\beta_R) \\ &= \int_{\beta_R=0}^{0.5} \left(\kappa_0(\beta_S,\beta_R) + \kappa_0(\beta_S, 1-\beta_R)\right) dG_R(\beta_R). \quad (44) \end{aligned}$$

Consider $\beta_R < 0.5$ such that $1-\beta_R > \beta_S > \beta_R$. For such β_R it holds

$$\begin{aligned} & \kappa_0(\beta_S,\beta_R) + \kappa_0(\beta_S, 1-\beta_R) \\ = & \left(\widetilde{\beta}(0,\beta_S) - \widetilde{\beta}(0,\beta_R)\right) - (\beta_S - \beta_R) \\ & + \left(\widetilde{\beta}(0,1-\beta_R) - \widetilde{\beta}(0,\beta_S)\right) - (1-\beta_R-\beta_S) \\ = & \widetilde{\beta}(0,1-\beta_R) - \widetilde{\beta}(0,\beta_R) + 2\beta_R - 1 \\ = & \frac{(1-\beta_R)(1-p)}{(1-\beta_R)(1-p)+\beta_R p} - \frac{\beta_R(1-p)}{\beta_R(1-p)+(1-\beta_R)p} + 2\beta_R - 1 \\ = & -\frac{(1-2p)^2(1-\beta_R)(1-2\beta_R)\beta_R}{(1-p+\beta_R(2p-1))(\beta_R+p(1-2\beta_R))} < 0. \end{aligned}$$

Since the probability mass of $\beta_R < 0.5$ such that the condition $1-\beta_R > \beta_S > \beta_R$ is satisfied is sufficiently large for β_S sufficiently close to 0.5, the right-hand side of (44) is negative as well. Hence, the sender would prefer to disclose 0-signal over no disclosure. The same claim for 1-signals follows by symmetry considerations. Consequently, the FD-equilibrium exists. ■

Appendix V: Proposition 2

Step 1. First, note that $\Delta(0, \beta_S, \beta_R)$ and $\Delta(1, \beta_S, \beta_R)$ are V-shaped with respect to either β_S or β_R reaching its minimum at $\beta_S = \beta_R$. Indeed, since $\widetilde{\beta}_i(0)$ is increasing in β_i, it follows that $\Delta(0, \beta_i, \beta_j)$ decreases in β_i if $\beta_i < \beta_j$ and increases in β_i otherwise, being equal to 0 for $\beta_i = \beta_j$. The same argument applies to $\Delta(1, \beta_S, \beta_R)$.

Step 2. Let us show another auxiliary result that $E_S[\Delta^{D0}(\varnothing, \beta_S, \beta_R)]$ and $E_S[\Delta^{D1}(\varnothing, \beta_S, \beta_R)]$ are V-shaped with respect to β_R reaching its minimum at $\beta_S = \beta_R$. Consider $E_S[\Delta^{D1}(\varnothing, \beta_S, \beta_R)]$. Using the expressions from Appendix I, we get:

$$\begin{aligned} & E_S[\Delta^{D1}(\varnothing, \beta_S, \beta_R)] \\ = \ & \frac{p(1-p\varphi) - \beta_S(2p-1)(1-\varphi)}{p - \beta_S(2p-1)} \frac{|\beta_S - \beta_R|}{1 - \varphi(1 - p + \beta_R(2p-1))}. \end{aligned} \tag{45}$$

Taking the derivative with respect to β_R and simplifying we obtain (for $\beta_R \neq \beta_S$)

$$\begin{aligned} & \frac{\partial E_S[\Delta^{D1}(\varnothing, \beta_S, \beta_R)]}{\partial \beta_R} \\ = \ & \operatorname{sgn}[\beta_R - \beta_S] \frac{p(1-p\varphi) - \beta_S(2p-1)(1-\varphi)}{p - \beta_S(2p-1)} \frac{1 - \varphi(1 - p + \beta_S(2p-1))}{(1 - \varphi(1 - p + \beta_R(2p-1)))^2} \end{aligned}$$

It is easy to verify that all terms on the right-hand side following the sign function are always positive. Hence, the sign of the derivative is determined by $\operatorname{sgn}[\beta_R - \beta_S]$, which implies that function $E_S[\Delta^{D1}(\varnothing, \beta_S, \beta_R)]$ is V-shaped with respect to β_R, being kinked at $\beta_S = \beta_R$ where it is equal to 0 (see 45).

Consider $E_S[\Delta^{D0}(\varnothing, \beta_S, \beta_R)]$. Using the expressions from Appendix I, we get:

$$\begin{aligned} & E_S[\Delta^{D0}(\varnothing, \beta_S, \beta_R)] \\ = \ & \frac{1 - p + \beta_S(2p-1)(1-\varphi) - (1-p)^2\varphi}{1 - p + \beta_S(2p-1)} \frac{|\beta_S - \beta_R|}{1 - \varphi(p - \beta_R(2p-1))}. \end{aligned} \tag{46}$$

Taking the derivative with respect to β_R and simplifying we obtain (for $\beta_R \neq \beta_S$)

$$\begin{aligned} & \frac{\partial E_S[\Delta^{D0}(\varnothing, \beta_S, \beta_R)]}{\partial \beta_R} \\ = \quad & \text{sgn}[\beta_R - \beta_S] \frac{1 - p + \beta_S(2p-1)(1-\varphi) - (1-p)^2\varphi}{1 - p + \beta_S(2p-1)} \frac{1 - \varphi(p - \beta_S(2p-1))}{(1 - \varphi(p - \beta_R(2p-1)))^2} \end{aligned}$$

It is easy to verify that all terms on the right-hand side following the sign function are always positive. Hence, the sign of the derivative is determined by $\text{sgn}[\beta_R - \beta_S]$, which again implies that function $E_S[\Delta^{D0}(\varnothing, \beta_S, \beta_R)]$ is V-shaped with respect to β_R, being kinked at $\beta_S = \beta_R$ where it is equal to 0 (see 46).

Step 3. Let us show that $E_S[\Delta^{FD}]$, $E_S[\Delta^{D0}]$, and $E_S[\Delta^{D1}]$ are all V-shaped with respect to β_R and reach their minimum at $\beta_S = \beta_R$. We have

$$\begin{aligned} E_S[\Delta^{FD}] &= \Pr[\sigma_S = 1|\beta_S]\Delta(1, \beta_S, \beta_R) \\ + \Pr[\sigma_S &= 0|\beta_S]\Delta(0, \beta_S, \beta_R) + \Pr[\sigma_S = \varnothing|\beta_S]|\beta_S - \beta_R|, \quad (47) \\ E_S[\Delta^{D0}] &= \Pr[\sigma_S = 0|\beta_S]\Delta(0, \beta_S, \beta_R) \\ & + (1 - \Pr[\sigma_S = 0|\beta_S])\Delta^{D0}(\varnothing, \beta_S, \beta_R), \quad (48) \\ E_S[\Delta^{D1}] &= \Pr[\sigma_S = 1|\beta_S]\Delta(1, \beta_S, \beta_R) \\ & + (1 - \Pr[\sigma_S = 1|\beta_S])\Delta^{D1}(\varnothing, \beta_S, \beta_R). \quad (49) \end{aligned}$$

Note now that by Steps 1 and 2, it holds true that $\Delta(0, \beta_S, \beta_R)$, $\Delta(1, \beta_S, \beta_R)$, $\Delta^{D0}(\varnothing, \beta_S, \beta_R)$, $\Delta^{D1}(\varnothing, \beta_S, \beta_R)$ and $|\beta_S - \beta_R|$ are all V-shaped and reach their minimum (which equals 0) for $\beta_S = \beta_R$. It follows immediately that $E_S[\Delta^{FD}], E_S[\Delta^{D0}]$ and $E_S[\Delta^{D1}]$ exhibit these same properties.

Step 4. Let us show that $E_S[\Delta]$ is uniquely defined, i.e. that if X and X' are two equilibrium disclosure rules given β_S, β_R, then $E_S[\Delta^X|\beta_S, \beta_R] = E_S[\Delta^{X'}|\beta_S, \beta_R]$. First note that $E_S[\Delta] = 0$ in any equilibrium if $\beta_S = \beta_R$ (see the proof of Lemma II.C). Consider $\beta_S \neq \beta_R$. Then, by Proposition 1 the only instances where the equilibrium is not unique are when $\beta_S = \beta_S^*(\beta_R)$ and $\beta_S = \beta_S^{**}(\beta_R)$. Consider $\beta_S = \beta_S^*(\beta_R)$, in which case by Proposition 1 there exist FD, D0 and mixed disclosure equilibria. By (21)-

(25) in the proof of Lemma II.G, if $\beta_S = \beta_S^*(\beta_R)$, then S must be indifferent between disclosing $\sigma = 1$ and non-disclosure for any δ_R^1, i.e. in any disclosure equilibrium in which $\sigma = 1$ is not disclosed with positive probability (this includes D0). I.e. it must be true that:

$$\Delta(1, \beta_S^*(\beta_R), \beta_R) = \Delta^{D0}(\varnothing, \beta_S^*(\beta_R), \beta_R). \tag{50}$$

Note that by (4) $\Delta^{D0}(\varnothing, \beta_S^*(\beta_R), \beta_R)$ is a weighted average between $\Delta(1, \beta_S^*(\beta_R), \beta_R)$ and $|\beta_S^*(\beta_R) - \beta_R|$. Together with (50), this implies

$$\Delta(1, \beta_S^*(\beta_R), \beta_R) = |\beta_S^*(\beta_R) - \beta_R|. \tag{51}$$

(47), (48), (50) and (51) jointly imply that $E_S[\Delta^{FD}|\beta_S = \beta_S^*(\beta_R)]$ is equal to $E_S[\Delta^{D0}|\beta_S = \beta_S^*(\beta_R)]$, as well as to the corresponding value under any other equilibrium involving randomization between disclosure and non-disclosure when $\sigma = 1$ (recall that this is the only possible mixed-disclosure strategy equilibrium if $\beta_S = \beta_S^*(\beta_R)$ by the proof of Lemma II.G). Consequently, $E_S[\Delta]$ is uniquely defined if $\beta_S = \beta_S^*(\beta_R)$. By an analogous argument, $E_S[\Delta]$ is uniquely defined if $\beta_S = \beta_S^{**}(\beta_R)$.

Step 5. Let us show that $E_S[\Delta]$ is continuous in β_R. Note first that $E_S[\Delta^{D0}]$, $E_S[\Delta^{FD}]$ and $E_S[\Delta^{D1}]$ are all continuous in β_R. Besides, by Step 4, $E_S[\Delta]$ is uniquely defined. This together with Proposition 1 implies that $E_S[\Delta]$ is equal either to $E_S[\Delta^{D0}]$, $E_S[\Delta^{FD}]$ or $E_S[\Delta^{D1}]$ depending on whether, respectively, $\beta_S \in (0, \beta_S^*(\beta_R)]$, $\beta_S \in [\beta_S^*(\beta_R), \beta_S^{**}(\beta_R)]$ and $\beta_S \in [\beta_S^{**}(\beta_R), 1)$, being continuous at $\beta_R = (\beta_S^*)^{-1}(\beta_S)$ and $\beta_R = (\beta_S^{**})^{-1}(\beta_S)$. Consequently, $E_S[\Delta]$ is also continuous in β_R.

Step 6. Consider finally the perceived disagreement from S's perspective. By Step 5, $E_S[\Delta]$ is continuous in β_R and is equal either to $E_S[\Delta^{D0}]$, $E_S[\Delta^{FD}]$ or $E_S[\Delta^{D1}]$. By Step 3, all these functions are V-shaped with respect to β_R reaching its minimum at $\beta_S = \beta_R$. Consequently, the same holds for $E_S[\Delta]$. ■

Appendix VI: Proposition 6

Proposition 6 follows from a set of Lemmas (Lemmas V.A to V.D), which are stated and proved in what follows. In a given SDE featuring the non-disclosure interval (s_1, s_2), we denote R's perceived disagreement conditional on disclosure of a signal s by $\Delta(s)$ and conditional on non-disclosure by $\Delta^{(s_1,s_2)}(\varnothing)$:

$$\begin{aligned} \Delta(s) &= \left|\widetilde{\beta}_S(s) - \widetilde{\beta}_R(s)\right| \text{ for } s \in [\underline{s}, \bar{s}], \\ \Delta^{(s_1,s_2)}(\varnothing) &= \left|E_R^{s_1,s_2}[\widetilde{\beta}_S|\varnothing] - \widetilde{\beta}_R^{s_1,s_2}(\varnothing)\right|. \end{aligned}$$

Lemma V.A *If $\beta_S \neq \beta_R$, then $\Delta(s)$ satisfies the following:*

i) $\lim_{s\to\underline{s}} \Delta(s) = \lim_{s\to\bar{s}} \Delta(s) = 0$.

ii) There exists $\widehat{s}$ such that $\Delta(s)$ is increasing in s for all $s < \widehat{s}$ and decreasing in s for all $s > \widehat{s}$.

iii) $\widetilde{s} > (<)\widehat{s}$ if and only if the player with the lower prior is less (more) confident. Instead, $\widetilde{s} = \widehat{s}$ if and only if $\beta_S = 1 - \beta_R$, i.e. if players are equally confident.

Proof.

Step 1. i) is immediate. To show ii) we first prove that there is a unique $\widehat{s}$ such that

$$\frac{d}{ds}\left(\widetilde{\beta}_S(\widehat{s}) - \widetilde{\beta}_R(\widehat{s})\right) = 0$$

Indeed,

$$\begin{aligned} &\frac{d}{ds}\left(\widetilde{\beta}_S(s) - \widetilde{\beta}_R(s)\right) \\ = &\frac{d}{ds}\left(\frac{\beta_S}{\beta_S + (1-\beta_S)\frac{f(s|0)}{f(s|1)}} - \frac{\beta_R}{\beta_R + (1-\beta_R)\frac{f(s|0)}{f(s|1)}}\right) \\ = &\frac{f(s|0)}{f(s|1)}\left(\frac{\beta_R(1-\beta_R)}{\left(\beta_R + (1-\beta_R)\frac{f(s|0)}{f(s|1)}\right)^2} - \frac{\beta_S(1-\beta_S)}{\left(\beta_S + (1-\beta_S)\frac{f(s|0)}{f(s|1)}\right)^2}\right)\frac{d}{ds}. \end{aligned} \quad (52)$$

Consider the solution to

$$\beta_R (1-\beta_R) \left(\beta_S + (1-\beta_S) \frac{f(s|0)}{f(s|1)}\right)^2 = \beta_S (1-\beta_S) \left(\beta_R + (1-\beta_R) \frac{f(s|0)}{f(s|1)}\right)^2.$$

Both sides are decreasing in s, but we claim that they increase at different rates. To see this, note that

$$\begin{aligned}
&\frac{d}{ds}\beta_R (1-\beta_R) \left(\beta_S + (1-\beta_S) \frac{f(s|0)}{f(s|1)}\right)^2 \\
= \quad & 2\beta_R (1-\beta_R)(1-\beta_S) \left(\beta_S + (1-\beta_S) \frac{f(s|0)}{f(s|1)}\right) \frac{d}{ds}\frac{f(s|0)}{f(s|1)}, \\
&\frac{d}{ds}\beta_S (1-\beta_S) \left(\beta_R + (1-\beta_R) \frac{f(s|0)}{f(s|1)}\right)^2 \\
= \quad & 2\beta_S (1-\beta_R)(1-\beta_S) \left(\beta_R + (1-\beta_R) \frac{f(s|0)}{f(s|1)}\right) \frac{d}{ds}\frac{f(s|0)}{f(s|1)}.
\end{aligned}$$

The result then follows from the fact that

$$\begin{aligned}
& 2\beta_R (1-\beta_R)(1-\beta_S) \left(\beta_S + (1-\beta_S) \frac{f(s|0)}{f(s|1)}\right) \frac{d}{ds}\frac{f(s|0)}{f(s|1)} \\
\gtreqless \quad & 2\beta_S (1-\beta_R)(1-\beta_S) \left(\beta_R + (1-\beta_R) \frac{f(s|0)}{f(s|1)}\right) \frac{d}{ds}\frac{f(s|0)}{f(s|1)}
\end{aligned}$$

is equivalent to

$$\beta_R\beta_S + \beta_R (1-\beta_S) \frac{f(s|0)}{f(s|1)} \lesseqgtr \beta_R\beta_S + \beta_S (1-\beta_R) \frac{f(s|0)}{f(s|1)}$$

which, in turn, is equivalent to $\beta_R \lesseqgtr \beta_S$. Hence, $\widehat{s}$ (where $\Delta(s)$ reaches its extremum) must be unique. Then, claim ii) follows from continuity and (i) together with $\Delta(\widetilde{s}) = |\beta_S - \beta_R| > 0$.

Step 2. To show (iii), define again $\overline{\beta} = \max\{\beta_S, \beta_R\}$ and $\underline{\beta} = \min\{\beta_S, \beta_R\}$ such that $\Delta(s) = \widetilde{\beta}(s, \overline{\beta}) - \widetilde{\beta}(s, \underline{\beta})$. From (52) we then have:

$$\begin{aligned}
\frac{d}{ds}\Delta(\widetilde{s}) \quad &= \quad \left(\underline{\beta}\left(1-\underline{\beta}\right) - \overline{\beta}\left(1-\overline{\beta}\right)\right) \frac{d}{ds}\frac{f(\widetilde{s}|0)}{f(\widetilde{s}|1)} \gtreqless 0 \\
&\iff \underline{\beta}\left(1-\underline{\beta}\right) \lesseqgtr \overline{\beta}\left(1-\overline{\beta}\right),
\end{aligned}$$

so that by claim ii) $\widetilde{s} > (<)\widehat{s}$ if and only if $\underline{\beta}$ is less (more) confident than $\overline{\beta}$, and $\widetilde{s} = \widehat{s}$ if and only if players are equally confident. ■

Lemma V.B *(i) If* $\beta_S = \{\beta_R, 1-\beta_R\}$*, then there exists an FD-equilibrium.*

(ii) If $\beta_S \neq \{\beta_R, 1-\beta_R\}$*, then in any equilibrium a positive measure of signals is not disclosed.*

Proof.

Step 1. Let us show the existence of FD for $\beta_S = \{\beta_R, 1-\beta_R\}$. If $\beta_S = \beta_R$ then trivially $\Delta(s) = 0$ for any s, so that S has always an incentive to disclose s. If $\beta_S = 1-\beta_R$, then $\widetilde{s} = \widehat{s}$ by Lemma V.A(iii). Consequently, for any $s \in [\underline{s}, \bar{s}]$ we obtain

$$\Delta^{FD}(\varnothing) = |\beta_S - \beta_R| = \Delta(\widetilde{s}) = \Delta(\widehat{s}) \geq \Delta(s),$$

where the last inequality is by Lemma V.A (ii). Hence, S has an incentive to disclose all signals in equilibrium.

Step 2. Let us show that for $\beta_S \neq \{\beta_R, 1-\beta_R\}$ there exists no equilibrium where the set of non-disclosed signals has 0-measure. Assume by contradiction that this is the case. Consider thus a putative equilibrium featuring a disclosure rule $\widetilde{D}$ such that the set of non-disclosed signals has 0-measure. Then, the perceived disagreement upon non-disclosure is $\Delta^{\widetilde{D}}(\varnothing) = |\beta_S - \beta_R|$, since R assigns probability 1 to the fact that S is uninformed. At the same time, since $\Delta(s)$ is single peaked at $\widehat{s}$ by Lemma V.A(ii) and $\widetilde{s} \neq \widehat{s}$ by Lemma V.A(iii), we have

$$\Delta(\widehat{s}) > \Delta(\widetilde{s}) = |\beta_S - \beta_R| = \Delta^{\widetilde{D}}(\varnothing),$$

so that S has an incentive not to disclose all signals located sufficiently close to $\widehat{s}$, which is a contradiction.

Lemma V.C *If* $\beta_S \neq \{\beta_R, 1-\beta_R\}$*, then the unique equilibrium is an SDE.*

Proof.

Step 0. Steps 1-2 introduce key equilibrium conditions. In steps 3-4, we show that there exists a unique SDE. Step 5 proves that any equilibrium is an SDE.

In what follows, we assume $\beta_S > \beta_R$. The proof for the reverse case follows the same steps and is omitted.

Step 1. Consider a putative simple disclosure equilibrium with non-disclosure interval (s_1, s_2). From R's point of view, S does not disclose an observed signal with probability

$$\Pr_R(s \in (s_1, s_2)) = \beta_R \int_{s_1}^{s_2} f(s|1)ds + (1-\beta_R)\int_{s_1}^{s_2} f(s|0)ds.$$

When S does not disclose, R's posterior is

$$\begin{aligned}
&\widetilde{\beta}_R^{s_1,s_2}(\varnothing)\\
= \; & \frac{\varphi}{(1-\varphi)+\varphi\Pr_R(s\in(s_1,s_2))}\int_{s_1}^{s_2}\left(\beta_R f(s|1)+(1-\beta_R)f(s|0)\right)\widetilde{\beta}_R(s)ds\\
&+\frac{(1-\varphi)}{(1-\varphi)+\varphi\Pr_R(s\in(s_1,s_2))}\beta_R
\end{aligned}$$

Similarly, R's belief about S's posterior in this case is

$$\begin{aligned}
&E_R^{s_1,s_2}[\widetilde{\beta}_S|\varnothing]\\
= \; & \frac{\varphi}{(1-\varphi)+\varphi\Pr_R(s\in(s_1,s_2))}\int_{s_1}^{s_2}\left(\beta_R f(s|1)+(1-\beta_R)f(s|0)\right)\widetilde{\beta}_S(s)ds\\
&+\frac{(1-\varphi)}{(1-\varphi)+\varphi\Pr_R(s\in(s_1,s_2))}\beta_S.
\end{aligned}$$

Step 2. Given the definition of SDE and the fact that $\Delta(s)$ is single peaked, S must be indifferent between disclosure and non-disclosure at s_1 and s_2. Hence, we require

$$\Delta(s) = \Delta^{(s_1,s_2)}(\varnothing) \text{ for } s = s_1, s_2. \tag{53}$$

Next, implicitly define $s_2^*(s_1)$ as a value of $s_2 \neq s_1$ equalizing

$$\Delta(s_1) = \Delta(s_2)$$

for given $s_1 < \widehat{s}$. There is a unique such value by Lemma V.A (ii). We additionally (abusively) define $s_2^*(\widehat{s}) = \widehat{s}$. Then, the equilibrium condition (53) holds if and only if

$$\Delta(s_1) = \Delta^{(s_1, s_2^*(s_1))}(\varnothing).$$

Define the function

$$\gamma(s_1) \equiv \Delta^{(s_1, s_2^*(s_1))}(\varnothing) - \Delta(s_1).$$

It follows that the SDE featuring the non-disclosure interval $(s_1, s_2^*(s_1))$ exists if and only if

$$\gamma(s_1) = 0. \tag{54}$$

In the next steps, we show that there always exists a unique value of s_1 satisfying the above condition, which implies that there always exists a unique SDE.

Step 3. This step proves existence of an SDE, i.e. show that there exists s_1 such that $\gamma(s_1) = 0$. Denote s_1' the smallest value of s such that $\Delta(s) = \Delta(\widehat{s}) = \beta_S - \beta_R$. Note that $s_1' < \widehat{s}$ given $\beta_S \neq \{\beta_R, 1 - \beta_R\}$. Indeed, we know from Lemma V.B. that in this case $\Delta(\widehat{s}) > \beta_S - \beta_R$, and we also know from Lemma V.A that $\Delta(s)$ is single peaked in s with a maximum at $\widehat{s}$ and with $\lim_{s \to \underline{s}} \Delta(s) = \lim_{s \to \bar{s}} \Delta(s) = 0$.

Let us prove that $\gamma(s_1') > 0$. By Step 1 we have

$$\begin{aligned}
\Delta^{(s_1', s_2^*(s_1'))}(\varnothing) &= \frac{\varphi}{(1-\varphi) + \varphi \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]} \int_{s_1'}^{s_2^*(s_1')} \left(\beta_S(s) - \beta_R(s)\right) \widetilde{f}(s) ds \\
&\quad + \left(1 - \frac{\varphi \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]}{(1-\varphi) + \varphi \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]}\right) (\beta_S - \beta_R) \\
&= \frac{\varphi}{(1-\varphi) + \varphi \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]} \int_{s_1'}^{s_2^*(s_1')} \left(\beta_S(s) - \beta_R(s)\right) \widetilde{f}(s) ds \\
&\quad + \left(1 - \frac{\varphi \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]}{(1-\varphi) + \varphi \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]}\right) \Delta(s_1') \\
&> \Delta(s_1')
\end{aligned} \tag{55}$$

where the second equality is by construction of s_1', and the strict inequality follows from the fact that $\beta_S(s) - \beta_R(s) > \Delta(s_1')$ for all $s \in (s_1', s_2^*(s_1'))$ since $s_1' < \widehat{s}$ as noted above. This implies that $\gamma(s_1') = \Delta^{(s_1', s_2^*(s_1'))}(\varnothing) - \Delta(s_1') > 0$.

Now let us show that $\gamma(\widehat{s}) < 0$. Since $\widehat{s} = s_2^*(\widehat{s})$ by construction, it holds that $\Pr_R\left[s \in (\widehat{s}, s_2^*(\widehat{s}))\right] = 0$ so that

$$\Delta^{(\widehat{s}, s_2^*(\widehat{s}))}(\varnothing) = \beta_S - \beta_R = \Delta(s_1') < \Delta(\widehat{s}), \tag{56}$$

where the inequality is due to $s_1' < \widehat{s}$.

Thus, we have shown that $\gamma(s_1') > 0$ and $\gamma(\widehat{s}) < 0$. From continuity of $\gamma(s)$ on $[s_1', \widehat{s}]$, it then follows that there exists at least one $s_1 \in (s_1', \widehat{s})$ such that $\gamma(s_1) = 0$.

Step 4. We now show that there exists a unique SDE. By Step 1 we have

$$\begin{aligned}
& \Delta^{(s_1, s_2^*(s_1))}(\varnothing) \\
= & \frac{\varphi}{(1-\varphi) + \varphi \Pr_R(s \in (s_1, s_2^*(s_1)))} \int_{s_1}^{s_2^*(s_1)} \left(\beta_S(s) - \beta_R(s)\right) \widetilde{f}(s) ds \\
& + \left(1 - \frac{\varphi \Pr_R(s \in (s_1, s_2^*(s_1)))}{(1-\varphi) + \varphi \Pr_R(s \in (s_1, s_2^*(s_1)))}\right) (\beta_S - \beta_R) \\
= & \frac{\varphi}{(1-\varphi) + \varphi \Pr_R(s \in (s_1, s_2^*(s_1)))} \begin{pmatrix} \int_{s_1}^{s_2^*(s_1)} \left(\beta_S(s) - \beta_R(s)\right) \widetilde{f}(s) ds \\ - \Pr_R\left[s \in (s_1, s_2^*(s_1))\right] (\beta_S - \beta_R) \end{pmatrix} \\
& + (\beta_S - \beta_R).
\end{aligned}$$

Denote $\eta(s_1) = \frac{\varphi}{(1-\varphi) + \varphi \Pr_R\left[s \in (s_1, s_2^*(s_1)\right]}$ so that

$$\begin{aligned}
\eta'(s_1) &= \frac{\partial \eta(s_1)}{\partial s_1} = -\left(\frac{\varphi}{(1-\varphi) + \varphi \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]}\right)^2 \\
& \quad \times \left(\frac{\partial \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]}{\partial s_1} + \frac{\partial \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right]}{\partial s_2^*} \frac{\partial s_2^*(s_1)}{\partial s_1}\right) \\
&= \left[\eta(s_1)\right]^2 \left(\widetilde{f}(s_1) - \widetilde{f}(s_2^*) \frac{\partial s_2^*}{\partial s_1}\right) > 0.
\end{aligned}$$

Then, taking the derivative of $\Delta^{(s_1,s_2^*(s_1))}(\varnothing)$ with respect to s_1 we obtain

$$\begin{aligned}
& \frac{\partial \Delta^{(s_1,s_2^*(s_1))}(\varnothing)}{\partial s_1} \\
= \; & \eta'(s_1)\left(\int_{s_1}^{s_2^*}(\beta_S(s)-\beta_R(s))\widetilde{f}(s)ds - \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right](\beta_S - \beta_R)\right) \\
& +\eta(s_1)(-(\beta_S(s_1)-\beta_R(s_1))\widetilde{f}(s_1) + (\beta_S(s_2^*)-\beta_R(s_2^*))\widetilde{f}(s_2^*)\frac{\partial s_2^*}{\partial s_1} \\
& +(\beta_S-\beta_R)\left(\widetilde{f}(s_1) - \widetilde{f}(s_2^*)\frac{\partial s_2^*}{\partial s_1}\right) \\
= \; & [\eta(s_1)]^2\left(\widetilde{f}(s_1) - \widetilde{f}(s_2^*)\frac{\partial s_2^*}{\partial s_1}\right) \\
& \times\left(\int_{s_1}^{s_2^*}(\beta_S(s)-\beta_R(s))\widetilde{f}(s)ds - \Pr_R\left[s \in (s_1', s_2^*(s_1'))\right](\beta_S - \beta_R)\right) \\
& -\eta(s_1)((\beta_S(s_1)-\beta_R(s_1))\left(\widetilde{f}(s_1) - \widetilde{f}(s_2^*)\frac{\partial s_2^*}{\partial s_1}\right) \\
& +(\beta_S-\beta_R)\left(\widetilde{f}(s_1) - \widetilde{f}(s_2^*)\frac{\partial s_2^*}{\partial s_1}\right) \\
= \; & \eta(s_1)\left(\widetilde{f}(s_1) - \widetilde{f}(s_2^*)\frac{\partial s_2^*}{\partial s_1}\right)\left(\begin{array}{c} \eta(s_1)\int_{s_1}^{s_2^*}(\beta_S(s)-\beta_R(s))\widetilde{f}(s)ds \\ +(1-\eta(s_1)\Pr_R\left[s \in (s_1', s_2^*(s_1'))\right])(\beta_S-\beta_R) \\ -(\beta_S(s_1)-\beta_R(s_1)) \end{array}\right) \\
= \; & \eta(s_1)\left(\widetilde{f}(s_1) - \widetilde{f}(s_2^*)\frac{\partial s_2^*}{\partial s_1}\right)\left(\Delta^{s_1,s_2^*(s_1)}(\varnothing) - \Delta(s_1)\right) \\
= \; & \eta(s_1)\left(\widetilde{f}(s_1) - \widetilde{f}(s_2^*)\frac{\partial s_2^*}{\partial s_1}\right)\gamma(s_1).
\end{aligned}$$

Thus,

$$\frac{\partial \gamma(s_1)}{\partial s_1} = \eta(s_1)\left(\widetilde{f}(s_1) - \widetilde{f}(s_2)\frac{\partial s_2^*}{\partial s_1}\right)\gamma(s_1) - \Delta'(s_1). \tag{57}$$

Note that $\Delta'(s_1) > 0$ and $\frac{\partial s_2^*}{\partial s_1} < 0$ for any $s_1 < \widehat{s}$ by Lemma V.A (ii). Then, (57) implies that for any $s_1 < \widehat{s}$ such that $\gamma(s_1) \leq 0$ it holds $\gamma'(s_1) < 0$. Consequently, if $\gamma(s') = 0$ for some $s' < \widehat{s}$, it is strictly decreasing for all $s_1 \in [s', \widehat{s})$. Hence, the equilibrium condition $\gamma(s_1) = 0$ can be satisfied for at most one value of $s_1 < \widehat{s}$.

Step 5. We prove by contradiction that any equilibrium is a simple disclosure equilibrium. Assume thus an equilibrium which is not an SDE. By Lemma V.B, the set of non-disclosed signals has a positive measure. Upon non-disclosure, let the perceived disagreement be denoted by $C > 0$. Conditional on obtaining a signal, S wants to disclose if and only if the resulting disagreement $\Delta(s)$ is smaller than C. Recall now that $\Delta(s)$ is single peaked at $\widehat{s}$ by Lemma V.A(ii). Hence, given that a positive measure of signals is not disclosed, we must have $C < \Delta(\widehat{s})$. Then, there are s_1, s_2 satisfying $\underline{s} < s_1 < s_2 < \overline{s}$ such that the actual disagreement is strictly higher than C after disclosing $s \in (s_1, s_2)$ and strictly lower than C after disclosing $s < s_1$ and $s > s_2$. In other words, this implies that for any putative equilibrium, there are s_1, s_2 satisfying $\underline{s} < s_1 < s_2 < \overline{s}$ such that S would strictly prefer not to disclose for $\sigma \in (s_1, s_2)$ and strictly prefer to disclose if $\sigma < s_1$ and $\sigma > s_2$. A putative equilibrium which is not an SDE thus gives rise to strict deviation incentives for S. ■

Lemma V.D

a) Assume that $\beta_S > \beta_R$. If $\beta_R < 1 - \beta_S$, i.e. R is more confident than S, then the equilibrium features $\widetilde{s} < s_1 < s_2$, i.e. all signals congruent with R's prior bias are disclosed. If $\beta_R > 1 - \beta_S$, i.e. R is less confident than S, then the equilibrium features $s_1 < s_2 < \widetilde{s}$, i.e. all signals congruent with S's prior bias are disclosed.

b) Assume that $\beta_S < \beta_R$. If $\beta_R > 1 - \beta_S$, i.e. R is more confident than S, then the equilibrium features $s_1 < s_2 < \widetilde{s}$, i.e. all signals congruent with R's prior bias are disclosed. If $\beta_R < 1 - \beta_S$, i.e. R is less confident than S, then the equilibrium features $\widetilde{s} < s_1 < s_2$, i.e. all signals congruent with S's prior bias are disclosed.

Proof.

We use the definitions of $\gamma(s_1)$, s_1' and $s_2^*(s_1)$ used in the proof of Lemma V.C (see Steps 2 and 3 there). By (56), $\gamma(\widehat{s}) < 0$. At the same time, by (55) we have that $\gamma(s_1') > 0$. Consequently, by the uniqueness of the SDE

$$s_1' < s_1. \tag{58}$$

Given that $\Delta(s)$ is single-peaked and the definition of s_2^*, this further implies

$$s_2 = s_2^*(s_1) < s_2^*(s_1'). \tag{59}$$

Now note that by construction $s_1' = \widetilde{s}$ if $\widetilde{s} < \widehat{s}$, and $s_2^*(s_1') = \widetilde{s}$ if $\widetilde{s} > \widehat{s}$. Then, the claims a) and b) follow by Lemma V.A(ii) together with (58) and (59). ■

Appendix VII: Propositions 7 and 8

Proof of Proposition 7

Step 0. We prove Point 1 in what follows. By assumption, it holds true that $s_1 < s_2 < \widetilde{s}$. By Lemmas V.C and V.D it follows that $\beta_R > 1 - \beta_S$. We focus on proving that S would strictly prefer to commit to full disclosure if $\beta_S > \beta_R$. Note that combining $\beta_R > 1 - \beta_S$ and $\beta_S > \beta_R$ implies $\beta_S > \frac{1}{2}$ and $\beta_R \in (1 - \beta_S, \beta_S)$. The proof that S instead prefers equilibrium disclosure given $\beta_S < \beta_R$ and $s_1 < s_2 < \widetilde{s}$ is briefly outlined in our final step. The proof of Point 2 is conceptually identical to that of Point 1 and thus entirely omitted.

Step 1. Assume that $\beta_S > \beta_R$. From S's perspective, the ex ante perceived disagreement in the SDE featuring thresholds $\{s_1, s_2\}$ is given by:

$$\begin{aligned}
&(1-\varphi)\left[E_R[\widetilde{\beta}_S^{s_1,s_2}|\varnothing] - \widetilde{\beta}_R^{s_1,s_2}(\varnothing)\right] \\
&+\varphi \int_{s_1}^{s_2} (\beta_S f(s|1) + (1-\beta_S) f(s|0))ds \left[E_R[\widetilde{\beta}_S^{s_1,s_2}|\varnothing] - \widetilde{\beta}_R^{s_1,s_2}(\varnothing)\right] \\
&+\varphi \int_{s_1}^{s_2} (\beta_S f(s|1) + (1-\beta_S) f(s|0))\Delta(s)ds.
\end{aligned}$$

Recall also that we know from Step 1 in the proof of Lemma V.C that

$$\begin{aligned}
E_R[\widetilde{\beta}_S^{s_1,s_2}|\varnothing] - \widetilde{\beta}_R^{s_1,s_2}(\varnothing) = &\frac{\varphi}{(1-\varphi) + \varphi \Pr_R(s \in (s_1, s_2))} \\
\times &\int_{s_1}^{s_2} (\beta_R f(s|1) + (1-\beta_R) f(s|0))\Delta(s)ds \\
+&\frac{(1-\varphi)}{(1-\varphi) + \varphi \Pr_R(s \in (s_1, s_2))} (\beta_S - \beta_R).
\end{aligned}$$

Step 2. We here consider a putative full disclosure equilibrium. From S's perspective, the ex ante perceived disagreement in an equilibrium with full disclosure is simply

$$\begin{aligned}&\varphi \int_{s_1}^{s_2} (\beta_S f(s|1) + (1-\beta_S) f(s|0))\Delta(s)ds \\ &+\varphi \int_{s_1}^{s_2} (\beta_S f(s|1) + (1-\beta_S) f(s|0))\Delta(s)ds \\ &+(1-\varphi)\left[\beta_S - \beta_R\right].\end{aligned}$$

Step 3. We introduce two expressions which we shall call Θ(Partial) and Θ(Full). These describe the expected perceived disagreement in S's eyes under each of the two disclosure rules, when restricting ourselves to those events where either $s \in [s_1, s_2]$ or S holds no signal (as otherwise the perceived disagreement is identical under the two regimes). We have:

$$\begin{aligned}&\Theta(\text{Partial}) \\ =\ & [\varphi \Pr_S(s \in (s_1, s_2)) + (1-\varphi)] \left[E_R[\widetilde{\beta}_S^{s_1,s_2}|\varnothing] - \widetilde{\beta}_R^{s_1,s_2}(\varnothing)\right] \\ =\ & [\varphi \Pr_S(s \in (s_1, s_2)) + (1-\varphi)] \\ & \times \left[\begin{array}{c} \frac{\varphi}{(1-\varphi)+\varphi \Pr_R(s\in(s_1,s_2))} \int_{s_1}^{s_2} (\beta_R f(s|1) + (1-\beta_R) f(s|0))\Delta(s)ds \\ + \frac{(1-\varphi)}{(1-\varphi)+\varphi \Pr_R(s\in(s_1,s_2))} (\beta_S - \beta_R) \end{array}\right]\end{aligned}$$

and

$$\Theta(\text{Full}) = \varphi \int_{s_1}^{s_2} \left[\beta_S f(s|1) + (1-\beta_S) f(s|0)\right] \Delta(s)ds + (1-\varphi)(\beta_S - \beta_R).$$

Our objective is to identify conditions under which Θ(Partial) $>$ Θ(Full), i.e.

$$\begin{aligned}& [\varphi \Pr_S(s \in (s_1, s_2)) + (1-\varphi)] \left[E_R[\widetilde{\beta}_S^{s_1,s_2}|\varnothing] - \widetilde{\beta}_R^{s_1,s_2}(\varnothing)\right] \\ >\ & \varphi \int_{s_1}^{s_2} \left[\beta_S f(s|1) + (1-\beta_S) f(s|0)\right] \Delta(s)ds + (1-\varphi)(\beta_S - \beta_R).\end{aligned}$$

Step 4. Define $\Pr_{\widehat{\beta}_R}(s \in (s_1, s_2))$ as the ex ante probability assigned $s \in [s_1, s_2]$, when using the prior $\widehat{\beta}_R$. I.e. let:

$$\Pr_{\widehat{\beta}_R}(s \in (s_1, s_2)) = \int_{s_1}^{s_2} (\widehat{\beta}_R f(s|1) + \left(1 - \widehat{\beta}_R\right) f(s|0)) ds.$$

We define $\Delta^{(s_1,s_2)}\left(\varnothing, \widehat{\beta}_R\right)$ as a slightly modified version of $E_R[\widetilde{\beta}_S^{s_1,s_2}|\varnothing] - \widetilde{\beta}_R^{s_1,s_2}(\varnothing)$, with the only difference that the distribution of signals is calculated based on the prior $\widehat{\beta}_R$. We let

$$\begin{aligned} & \Delta^{(s_1,s_2)}\left(\varnothing, \widehat{\beta}_R\right) \\ = & \frac{\varphi}{(1-\varphi) + \varphi \Pr_{\widehat{\beta}_R}(s \in (s_1, s_2))} \\ & \times \int_{s_1}^{s_2} (\widehat{\beta}_R f(s|1) + \left(1 - \widehat{\beta}_R\right) f(s|0)) \Delta(s) ds \\ & + \frac{(1-\varphi)}{(1-\varphi) + \varphi \Pr_{\widehat{\beta}_R}(s \in (s_1, s_2))} \left(\beta_S - \beta_R\right). \end{aligned}$$

Let us finally define

$$\widehat{\Theta}(\text{Partial}, \widehat{\beta}_R) = \left[\varphi \Pr_S(s \in (s_1, s_2)) + (1-\varphi)\right] \left[\Delta^{(s_1,s_2)}\left(\varnothing, \widehat{\beta}_R\right)\right]$$

and note that $\widehat{\Theta}(\text{Partial}, \beta_R) = \Theta(\text{Partial})$.

In what follows, we shall consider the value of the above function for $\widehat{\beta}_R = \beta_S$ and for $\widehat{\beta}_R \in (1 - \beta_S, \beta_S)$. We show in step 5 that $\widehat{\Theta}(\text{Partial}, \beta_S) = \Theta(\text{Full})$. We show in step 6 that for any $\widehat{\beta}_R \in (1 - \beta_S, \beta_S)$, we have $\widehat{\Theta}(\text{Partial}, \widehat{\beta}_R) > \Theta(\text{Full})$. Given that by assumption $\beta_R \in (1 - \beta_S, \beta_S)$, this implies that in particular $\widehat{\Theta}(\text{Partial}, \beta_R) = \Theta(\text{Partial}) > \Theta(\text{Full})$.

Step 5. Note that when setting $\widehat{\beta}_R = \beta_S$, we have:

$$\begin{aligned}
&\widehat{\Theta}(\text{Partial},\beta_S)\\
=\;& \left[\varphi \Pr_S(s\in(s_1,s_2))+(1-\varphi)\right]\left[\Delta^{(s_1,s_2)}\left(\varnothing,\beta_S\right)\right]\\
=\;& \left[\varphi \Pr_S(s\in(s_1,s_2))+(1-\varphi)\right]\\
&\times\left[\begin{array}{c}\frac{\varphi}{(1-\varphi)+\varphi\Pr_S(s\in(s_1,s_2))}\int_{s_1}^{s_2}\left(\beta_S f(s|1)+(1-\beta_S)f(s|0)\right)\Delta(s)ds\\ +\frac{(1-\varphi)}{(1-\varphi)+\varphi\Pr_S(s\in(s_1,s_2))}\left(\beta_S-\beta_R\right)\end{array}\right]\\
=\;& \varphi\int_{s_1}^{s_2}\left[\beta_S f(s|1)+(1-\beta_S)f(s|0)\right]\Delta(s)ds+(1-\varphi)\left(\beta_S-\beta_R\right)\\
=\;& \Theta(\text{Full}).
\end{aligned}$$

Step 6. Here, we show that $\Delta^{(s_1,s_2)}\left(\varnothing,\widehat{\beta}_R\right)$ increases (resp. decreases) as $\widehat{\beta}_R$ decreases (resp. increases), for $\widehat{\beta}_R\leq\beta_S$. Note that we can rewrite $\Delta^{(s_1,s_2)}\left(\varnothing,\widehat{\beta}_R\right)$ as follows:

$$\begin{aligned}
&\Delta^{(s_1,s_2)}\left(\varnothing,\widehat{\beta}_R\right)\\
=\;&\left[\begin{array}{c}\frac{\varphi\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}{(1-\varphi)+\varphi\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}\int_{s_1}^{s_2}\frac{\left(\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)\right)}{\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}\Delta(s)ds\\ +\frac{(1-\varphi)}{(1-\varphi)+\varphi\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}\left(\beta_S-\beta_R\right)\end{array}\right].
\end{aligned}$$

From the above expression, note that $\Delta^{(s_1,s_2)}\left(\varnothing,\widehat{\beta}_R\right)$ is thus a weighted average of the expressions

$$\begin{aligned}
&E_{\widehat{\beta}_R}\left[\widetilde{\beta}_S(s)-\widetilde{\beta}_R(s)\,|s\in[s_1,s_2]\right]\\
=\;&\int_{s_1}^{s_2}\frac{\left(\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)\right)}{\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}\Delta(s)ds
\end{aligned}$$

and $(\beta_S-\beta_R)$. The first expression is weighted by $\frac{\varphi\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}{(1-\varphi)+\varphi\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}$ and the second is weighted by $\frac{(1-\varphi)}{(1-\varphi)+\varphi\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}$. In other words, $\Delta^{(s_1,s_2)}\left(\varnothing,\widehat{\beta}_R\right)$ can be written as:

$$\Delta^{(s_1,s_2)}\left(\varnothing,\widehat{\beta}_R\right)=p(\widehat{\beta}_R)A(\widehat{\beta}_R)+(1-p(\widehat{\beta}_R))(\beta_S-\beta_R),$$

where we let

$$p(\widehat{\beta}_R) = \frac{\varphi \Pr_{\widehat{\beta}_R}(s \in (s_1, s_2))}{(1-\varphi) + \varphi \Pr_{\widehat{\beta}_R}(s \in (s_1, s_2))}$$

and we let

$$A(\widehat{\beta}_R) = E_{\widehat{\beta}_R}\left[\widetilde{\beta}_S(s) - \widetilde{\beta}_R(s)\,|s \in [s_1, s_2]\right].$$

The derivative of $\Delta^{(s_1,s_2)}\left(\varnothing, \widehat{\beta}_R\right)$ w.r.t. $\widehat{\beta}_R$ is thus given by

$$\begin{aligned}\frac{\partial \Delta^{(s_1,s_2)}\left(\varnothing, \widehat{\beta}_R\right)}{\partial \widehat{\beta}_R} &= \frac{\partial p(\widehat{\beta}_R)}{\partial \widehat{\beta}_R} A(\widehat{\beta}_R) + p(\widehat{\beta}_R)\frac{\partial A(\widehat{\beta}_R)}{\partial \widehat{\beta}_R} - \frac{\partial p(\widehat{\beta}_R)}{\partial \widehat{\beta}_R}(\beta_S - \beta_R) \\ &= p(\widehat{\beta}_R)\frac{\partial A(\widehat{\beta}_R)}{\partial \widehat{\beta}_R} + \frac{\partial p(\widehat{\beta}_R)}{\partial \widehat{\beta}_R}\left[A(\widehat{\beta}_R) - (\beta_S - \beta_R)\right].\end{aligned}$$

In order to prove that $\frac{\partial \Delta^{(s_1,s_2)}\left(\varnothing, \widehat{\beta}_R\right)}{\partial \widehat{\beta}_R} < 0$ for $\widehat{\beta}_R \in (1-\beta_S, \beta_S)$, it thus suffices to show that $\frac{\partial A(\widehat{\beta}_R)}{\partial \widehat{\beta}_R} < 0$,

$$\left[A(\widehat{\beta}_R) - (\beta_S - \beta_R)\right] > 0$$

and $\frac{\partial p(\widehat{\beta}_R)}{\partial \widehat{\beta}_R} < 0$. We show in what follows that these properties are indeed satisfied for $\widehat{\beta}_R \in (1-\beta_S, \beta_S)$.

Note first that $\frac{\partial \Pr_{\widehat{\beta}_R}(s \in (s_1,s_2))}{\partial \widehat{\beta}_R} = \int_{s_1}^{s_2}(f(s|1) - f(s|0))ds$, which is strictly negative given that we know that $f(s|0) > f(s|1)$ for any $s \in [s_1, s_2]$, recalling that $s_1 < s_2 < \widetilde{s}$ by assumption. It follows immediately that $\frac{(1-\varphi)}{(1-\varphi)+\varphi \Pr_{\widehat{\beta}_R}(s \in (s_1,s_2))} = 1 - p(\widehat{\beta}_R)$ increases in $\widehat{\beta}_R$ and that $\frac{\varphi \Pr_{\widehat{\beta}_R}(s \in (s_1,s_2))}{(1-\varphi)+\varphi \Pr_{\widehat{\beta}_R}(s \in (s_1,s_2))} = p(\widehat{\beta}_R)$ decreases in $\widehat{\beta}_R$. Second, to show that $A(\widehat{\beta}_R) - (\beta_S - \beta_R) > 0$ note that by the facts that $s_1 < s_2 < \widetilde{s}$, that $\Delta(s_1) = \Delta(s_2)$ in SDE and that $\Delta(s)$ is hum shaped in s, we obtain

$$\beta_S - \beta_R = \Delta(\widetilde{s}) < \Delta(s_1) < \Delta(s)|s \in (s_1, s_2).$$

Third, we now show that $A(\widehat{\beta}_R) = E_{\widehat{\beta}_R}\left[\left(\widetilde{\beta}_S(s) - \widetilde{\beta}_R(s)\right)|s \in [s_1, s_2]\right]$ decreases as $\widehat{\beta}_R$ increases.

Note that:

$$
\begin{aligned}
& \frac{\partial\left[\int_{s_1}^{s_2}\frac{(\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0))}{\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))}\Delta(s)ds\right]}{\partial\widehat{\beta}_R} \\
= & \int_{s_1}^{s_2}\frac{\begin{pmatrix} (f(s|1)-f(s|0))\left[\int_{s_1}^{s_2}\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)ds\right] \\ -\left[\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)\right]\left[\int_{s_1}^{s_2}(f(s|1)-f(s|0))ds\right]\end{pmatrix}}{\left[\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))\right]^2}\Delta(s)ds \\
= & \frac{\begin{pmatrix} \left[\int_{s_1}^{s_2}\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)ds\right]\left[\int_{s_1}^{s_2}(f(s|1)-f(s|0))\Delta(s)ds\right] \\ -\left[\int_{s_1}^{s_2}(f(s|1)-f(s|0))ds\right]\left[\int_{s_1}^{s_2}\left(\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)\right)\Delta(s)ds\right]\end{pmatrix}}{\left[\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))\right]^2} \\
= & \frac{\begin{pmatrix} -\left[\int_{s_1}^{s_2}(f(s|1)-f(s|0))ds\right]\left[\int_{s_1}^{s_2}\left(\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)\right)\Delta(s)ds\right] \\ +\left[\int_{s_1}^{s_2}\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)ds\right]\left[\int_{s_1}^{s_2}(f(s|1)-f(s|0))\Delta(s)ds\right]\end{pmatrix}}{\left[\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))\right]^2} \\
< & \frac{\begin{pmatrix} -\left[\int_{s_1}^{s_2}(f(s|1)-f(s|0))ds\right]\left[\int_{s_1}^{s_2}\left(\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)\right)\Delta(s)ds\right] \\ +\left[\int_{s_1}^{s_2}\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)ds\right]\left[\int_{s_1}^{s_2}(f(s|1)-f(s|0))\right]\left[\int_{s_1}^{s_2}\Delta(s)ds\right]\end{pmatrix}}{\left[\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))\right]^2} \\
= & \frac{-\left[\int_{s_1}^{s_2}(f(s|1)-f(s|0))ds\right]\begin{pmatrix} \left[\int_{s_1}^{s_2}\left(\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)\right)\Delta(s)ds\right] \\ -\left[\int_{s_1}^{s_2}\widehat{\beta}_R f(s|1)+\left(1-\widehat{\beta}_R\right)f(s|0)ds\right]\left[\int_{s_1}^{s_2}\Delta(s)ds\right]\end{pmatrix}}{\left[\Pr_{\widehat{\beta}_R}(s\in(s_1,s_2))\right]^2} \\
< & \ 0.
\end{aligned}
$$

Above, the first equality follows from the application of Leibniz' rule. The first and the second inequality follow from applying Hölder's inequality.

Thus, we have shown that $\frac{\partial\Delta^{(s_1,s_2)}\left(\varnothing,\widehat{\beta}_R\right)}{\partial\widehat{\beta}_R}<0$. This implies that

$$
\frac{\partial\widehat{\Theta}(\text{Partial},\widehat{\beta}_R)}{\partial\widehat{\beta}_R}<0.
$$

In sum, we obtain that for $\beta_S > \beta_R$ and $s_1 < s_2 < \widetilde{s}$ it holds

$$\Theta(\text{Partial}) = \widehat{\Theta}(\text{Partial},\beta_R) > \widehat{\Theta}(\text{Partial},\beta_S) = \Theta(\text{Full}). \tag{60}$$

Here, the inequality follows from the previous inequality, while the second equality is by Step 5.

Step 7. Suppose now instead that $\beta_S < \beta_R$ and $s_1 < s_2 < \widetilde{s}$. Note that combining the assumptions $\beta_S < \beta_R$ and $s_1 < s_2 < \widetilde{s}$ implies that $\beta_R \in (\beta_S, 1)$ by Lemma V.D. The argument follows the same logic as above. It still holds true $\widehat{\Theta}(\text{Partial},\beta_S) = \Theta(\text{Full})$ and that $\widehat{\Theta}(\text{Partial},\beta_R) = \Theta(\text{Partial})$. It also still holds true that $\Theta(\text{Partial},\widehat{\beta}_R)$ is decreasing in $\widehat{\beta}_R$. It follows that

$$\Theta(\text{Partial}) = \widehat{\Theta}(\text{Partial},\beta_R) < \widehat{\Theta}(\text{Partial},\beta_S) = \Theta(\text{Full}).$$

■

Proof of Proposition 8

The argument here is exactly identical to the proof of the counterpart of this result for the case of binary signals (Proposition 4). ■

Appendix VIII: Proposition 9

For convenience, we prove first point 3, then 2 and finally 1.

Proof of Proposition 9.3.a).

Step 1. We here prove that there are finite $\sigma' < \sigma''$ such that σ increases disagreement if and only if $\sigma \notin [\sigma', \sigma'']$. Recall that the perceived disagreement is given by

$$\Delta(d, \beta_S, \beta_R) = \left| E_R[E_S[\omega|\sigma] \,|d] - E_R[\omega|d] \right|. \tag{61}$$

Note that

$$E_i[\omega|\sigma] = \frac{\mu_i \frac{1}{\gamma_i^2} + \sigma \frac{1}{\gamma_\varepsilon^2}}{\frac{1}{\gamma_i^2} + \frac{1}{\gamma_\varepsilon^2}}.$$

Hence,

$$\Delta(\sigma) = |E_S[\omega|\sigma] - E_R[\omega|\sigma]| = |D(\sigma)|, \tag{62}$$

where

$$\begin{aligned} & D(\sigma) \\ = & \frac{\mu_S \frac{1}{\gamma_S^2} + \sigma \frac{1}{\gamma_\varepsilon^2}}{\frac{1}{\gamma_S^2} + \frac{1}{\gamma_\varepsilon^2}} - \frac{\mu_R \frac{1}{\gamma_R^2} + \sigma \frac{1}{\gamma_\varepsilon^2}}{\frac{1}{\gamma_R^2} + \frac{1}{\gamma_\varepsilon^2}} \\ = & -\frac{\gamma_\varepsilon^2}{(\gamma_R^2 + \gamma_\varepsilon^2)(\gamma_S^2 + \gamma_\varepsilon^2)} [\sigma (\gamma_R^2 - \gamma_S^2) - \gamma_R^2 \mu_S + \gamma_S^2 \mu_R + (\mu_R - \mu_S) \gamma_\varepsilon^2]. \end{aligned} \tag{63}$$

Note that $D(\sigma)$ is a linear function, so that it has a unique root in $\Re$. Consequently, $|D(\sigma)|$ is V-shaped in σ, with the minimum value of 0. It follows immediately that there exist $\sigma' < \sigma''$ such that $|D(\sigma)| > |\mu_S - \mu_R|$ (i.e. σ increases disagreement) if and only if $\sigma \notin [\sigma', \sigma'']$.

Step 2. Let us show that any equilibrium under $\gamma_S \neq \gamma_R$ and $\mu_S \neq \mu_R$ features a disclosure interval $[\underline{\sigma}, \overline{\sigma}]$ such that σ is disclosed if and only if $\sigma \in [\underline{\sigma}, \overline{\sigma}]$.

First, note that FD never exists under $\gamma_S \neq \gamma_R$ by Step 1, since otherwise S would have an incentive to deviate by concealing any signal σ such that $|D(\sigma)| > |\mu_S - \mu_R|$.

Second, note that an equilibrium with disclosure rule $\widetilde{D}$ must feature a positive and finite value of perceived disagreement conditional on no disclosure $\Delta^{\widetilde{D}}(\varnothing)$. The fact that $\Delta^{\widetilde{D}}(\varnothing)$ should be finite can be shown by contradiction. Suppose indeed that $\Delta^{\widetilde{D}}(\varnothing)$ is not finite. Then there would exist an FD-equilibrium, as S would always favour disclosing over not disclosing. But we know that there exists no FD-equilibrium, as stated above. The fact that $\Delta^{\widetilde{D}}(\varnothing)$ must be positive can also be shown by contradiction. If this is not the case, then in equilibrium all signals must be concealed other than the unique signal $\widetilde{\sigma}$ such that $D(\widetilde{\sigma}) = 0$. Then, R's posterior belief distribution will not change after no disclosure, while R would expect that S's posterior mean will be strictly between μ_R and μ_S.[32] Hence, $\Delta^{\widetilde{D}}(\varnothing) > 0$ which is a contradiction.

Consider thus an equilibrium with disclosure rule $\widetilde{D}$ featuring a positive and finite $\Delta^{\widetilde{D}}(\varnothing)$. In this case, every signal σ such that $\Delta(\sigma) \leq \Delta^{\widetilde{D}}(\varnothing)$ will be disclosed, and every signal such that $\Delta(\sigma) > \Delta^{\widetilde{D}}(\varnothing)$ will not be disclosed. Given that $\Delta^{\widetilde{D}}(\sigma)$ is V-shaped in σ, the claim follows. Next, by (62) and linearity of $D(\sigma)$ it follows that $\Delta(\sigma)$ is symmetric around $\widetilde{\sigma}$, where

$$\begin{aligned} \Delta(\widetilde{\sigma}) &= 0 \Leftrightarrow \\ D(\widetilde{\sigma}) &= 0 \Leftrightarrow \\ \widetilde{\sigma} &= \frac{\mu_S(\gamma_R^2+\gamma_\varepsilon^2)-\mu_R(\gamma_S^2+\gamma_\varepsilon^2)}{\gamma_R^2-\gamma_S^2}. \end{aligned} \tag{64}$$

Consequently, the disclosure interval (characterized by $\{\sigma : \Delta(\sigma) \leq \Delta^{\widetilde{D}}(\varnothing)\}$ for a given equilibrium $\Delta^{\widetilde{D}}(\varnothing)$) is also symmetric around $\widetilde{\sigma}$.

Step 3. This shows existence of an equilibrium of the type characterized in Step 2. Denote R's perceived disagreement conditional on disclosure in an equilibrium featuring disclosure interval $(\underline{\sigma}, \overline{\sigma})$ as $\Delta^{(\underline{\sigma},\overline{\sigma})}(\varnothing)$. We have

$$\begin{aligned} &\Delta^{(\underline{\sigma},\overline{\sigma})}(\varnothing) \\ = &\left| \tau\mu_S + (1-\tau)\begin{bmatrix} \int_{\sigma\leq\underline{\sigma}} E_S[\omega|\sigma]\widetilde{f}(\sigma\,|d=\varnothing,\underline{\sigma},\overline{\sigma})d\sigma \\ +\int_{\sigma\geq\overline{\sigma}} E_S[\omega|\sigma]\widetilde{f}(\sigma\,|d=\varnothing,\underline{\sigma},\overline{\sigma})d\sigma \end{bmatrix} \right. \\ &\left. -\tau\mu_R - (1-\tau)\begin{bmatrix} \int_{\sigma\leq\underline{\sigma}} E_R[\omega|\sigma]\widetilde{f}(\sigma\,|d=\varnothing,\underline{\sigma},\overline{\sigma})d\sigma \\ +\int_{\sigma\geq\overline{\sigma}} E_R[\omega|\sigma]\widetilde{f}(\sigma\,|d=\varnothing,\underline{\sigma},\overline{\sigma})d\sigma \end{bmatrix} \right| \\ = &\left| \tau(\mu_S-\mu_R) + (1-\tau)\begin{bmatrix} \int_{\sigma\leq\underline{\sigma}} (E_S[\omega|\sigma]-E_R[\omega|\sigma])\widetilde{f}(\sigma\,|d=\varnothing,\underline{\sigma},\overline{\sigma})d\sigma \\ +\int_{\sigma\geq\overline{\sigma}} (E_S[\omega|\sigma]-E_R[\omega|\sigma])\widetilde{f}(\sigma\,|d=\varnothing,\underline{\sigma},\overline{\sigma})d\sigma \end{bmatrix} \right|, \end{aligned}$$

where $\tau = P(\sigma = \varnothing\,|d=\varnothing,\underline{\sigma},\overline{\sigma})$, and $\widetilde{f}(\sigma\,|d=\varnothing,\underline{\sigma},\overline{\sigma})$ is the conditional distribution of σ given $d=\varnothing$ from the perspective of R, in an equilibrium featuring disclosure

[32] See Chapter 10 of Technical Appendix in Vives (2010).

interval $[\underline{\sigma}, \overline{\sigma}]$.

Given the V-shape of $\Delta(\sigma)$, a profile $\underline{\sigma}, \overline{\sigma}$ constitutes an equilibrium if and only if:

$$\Delta^{(\underline{\sigma},\overline{\sigma})}(\varnothing) = \Delta(\underline{\sigma}) = \Delta(\overline{\sigma}). \tag{65}$$

For any given $x > 0$, let $\underline{\sigma}(x), \overline{\sigma}(x)$ denote the unique pair of signals satisfying

$$\Delta(\underline{\sigma}(x)) = \Delta(\overline{\sigma}(x)) = x,$$

which exists for any $x > 0$ given that (63) is unbounded. Hence, the equilibrium condition (65) is equivalent to

$$\Delta^{(\underline{\sigma}(x),\overline{\sigma}(x))}(\varnothing) = x. \tag{66}$$

Note that for $x = 0$, i.e. if all signals are concealed, then it must be true that

$$\Delta^{(\underline{\sigma}(0),\overline{\sigma}(0))}(\varnothing) > 0 \tag{67}$$

(see Step 2).

Next, let us show that

$$\lim_{x\to\infty} \Delta^{(\underline{\sigma}(x),\overline{\sigma}(x))}(\varnothing) = |\mu_S - \mu_R| < x.$$

Note that

$$\Delta^{(\underline{\sigma}(x),\overline{\sigma}(x))}(\varnothing) = \left| P(\sigma = \varnothing \,|d = \varnothing, \underline{\sigma}(x), \overline{\sigma}(x)\,)(\mu_S - \mu_R) + \varsigma(\underline{\sigma}(x), \overline{\sigma}(x)) \right|,$$

where

$$\begin{aligned} &\varsigma(\underline{\sigma}(x), \overline{\sigma}(x)) \\ = \quad & P(\sigma \neq \varnothing \,|d = \varnothing, \underline{\sigma}(x), \overline{\sigma}(x)\,) \left[\begin{array}{c} \int_{\sigma \le \underline{\sigma}(x)} D(\sigma) \widetilde{f}(\sigma \,|d = \varnothing, \underline{\sigma}(x), \overline{\sigma}(x)\,) d\sigma \\ + \int_{\sigma \ge \overline{\sigma}(x)} D(\sigma) \widetilde{f}(\sigma \,|d = \varnothing, \underline{\sigma}(x), \overline{\sigma}(x)\,) d\sigma \end{array} \right]. \end{aligned}$$

Let us show that $\lim_{x\to\infty} \varsigma(\underline{\sigma}(x), \overline{\sigma}(x)) = 0$. We have

$$\begin{aligned}
\varsigma(\underline{\sigma}(x), \overline{\sigma}(x)) &= \frac{\varphi P(\sigma \notin [\underline{\sigma}(x), \bar{\sigma}(x)])}{\varphi P(\sigma \notin [\underline{\sigma}(x), \bar{\sigma}(x)]) + (1-\varphi)} \\
&\quad \times \begin{bmatrix} \int_{\sigma \leq \underline{\sigma}(x)} D(\sigma) \widetilde{f}(\sigma \,| d = \varnothing, \underline{\sigma}(x), \overline{\sigma}(x)) d\sigma \\ + \int_{\sigma \geq \overline{\sigma}(x)} D(\sigma) \widetilde{f}(\sigma \,| d = \varnothing, \underline{\sigma}(x), \overline{\sigma}(x)) d\sigma \end{bmatrix} \\
&= \frac{\varphi P(\sigma \notin [\underline{\sigma}(x), \bar{\sigma}(x)])}{\varphi P(\sigma \notin [\underline{\sigma}(x), \bar{\sigma}(x)]) + (1-\varphi)} \begin{bmatrix} \int_{\sigma \leq \underline{\sigma}(x)} D(\sigma) \frac{f(\sigma)}{P(\sigma \notin [\underline{\sigma}(x), \bar{\sigma}(x)])} d\sigma \\ + \int_{\sigma \geq \overline{\sigma}(x)} D(\sigma) \frac{f(\sigma)}{P(\sigma \notin [\underline{\sigma}(x), \bar{\sigma}(x)])} d\sigma \end{bmatrix} \\
&= \frac{\varphi}{\varphi P(\sigma \notin [\underline{\sigma}(x), \bar{\sigma}(x)]) + (1-\varphi)} \begin{bmatrix} \int_{\sigma \leq \underline{\sigma}(x)} D(\sigma) f(\sigma) d\sigma \\ + \int_{\sigma \geq \overline{\sigma}(x)} D(\sigma) f(\sigma) d\sigma \end{bmatrix}. \qquad (68)
\end{aligned}$$

Note that

$$\lim_{x\to\infty} \frac{\varphi}{\varphi P(\sigma \notin [\underline{\sigma}(x), \bar{\sigma}(x)]) + (1-\varphi)} = \frac{\varphi}{1-\phi}. \qquad (69)$$

At the same time, given that $E[D(\sigma)]$ must be finite (since σ is normally distributed and $D(\sigma)$ is linear in σ), we have

$$\begin{aligned}
& \lim_{x\to\infty} \int_{\sigma \leq \underline{\sigma}(x)} D(\sigma) f(\sigma) d\sigma \\
= & \lim_{x\to\infty} \left(E[D(\sigma)] - \int_{\sigma > \underline{\sigma}(x)} D(\sigma) f(\sigma) d\sigma \right) \\
= & E[D(\sigma)] - \lim_{x\to\infty} \int_{\underline{\sigma}(x)}^{\infty} D(\sigma) f(\sigma) d\sigma \\
= & E[D(\sigma)] - E[D(\sigma)] = 0. \qquad (70)
\end{aligned}$$

where the third equality is due to the fact that $\underline{\sigma}(x)$ must be linear in x (since its inverse function $\Delta(\sigma)$ is linear in σ). By the same argument,

$$\lim_{x\to\infty} \int_{\sigma \geq \overline{\sigma}(x)} D(\sigma) f(\sigma) d\sigma = 0. \qquad (71)$$

(68), (69), (70) and (71) together imply

$$\lim_{x\to\infty} \varsigma(\underline{\sigma}(x), \overline{\sigma}(x)) = 0. \qquad (72)$$

We may conclude that

$$\lim_{x\to\infty} \Delta^{(\underline{\sigma}(x),\overline{\sigma}(x))}(\varnothing) = |\mu_S - \mu_R| < x. \tag{73}$$

Given the continuity of

$$\Delta^{(\underline{\sigma}(x),\overline{\sigma}(x))}(\varnothing)$$

in x, it follows from (67) and (73) that as x increases from $\underline{x} = 0$ to $+\infty$, there is some $x \in (0, +\infty)$ such that the equilibrium condition (66) is satisfied. ■

Proof of Proposition 9.3.b).

Assume $\mu_R > \mu_S$ (the proof for $\mu_R < \mu_S$ proceeds symmetrically). We obtain

$$\begin{aligned} \mu_S - \widetilde{\sigma} &= \frac{(\mu_R - \mu_S)(\gamma_S^2 + \gamma_\varepsilon^2)}{\gamma_R^2 - \gamma_S^2}, \\ \mu_R - \widetilde{\sigma} &= \frac{(\mu_R - \mu_S)(\gamma_R^2 + \gamma_\varepsilon^2)}{\gamma_R^2 - \gamma_S^2}. \end{aligned}$$

Hence, if $\gamma_R > \gamma_S$ we get $\widetilde{\sigma} < \mu_S < \mu_R$, while if $\gamma_R < \gamma_S$ we get $\widetilde{\sigma} > \mu_R > \mu_S$. Thus, $\widetilde{\sigma} \notin [\mu_S, \mu_R]$ while $\widetilde{\sigma}$ is closer to the mean of the more confident player.

Finally, note that the Hausdorff distance between the disclosure interval $D = [\widetilde{\sigma} - \eta, \widetilde{\sigma} + \eta]$ and a mean μ_i is given by the largest distance between a point in D and μ_i. In case if $\gamma_R > \gamma_S$ so that $\widetilde{\sigma} < \mu_S < \mu_R$, the furthest point from either μ_S or μ_R is $\widetilde{\sigma} - \eta$, which is then closer to the prior of the more confident player μ_S. In case if $\gamma_R < \gamma_S$ so that $\widetilde{\sigma} > \mu_R > \mu_S$, the furthest point from either μ_S or μ_R is $\widetilde{\sigma} + \eta$, which is then closer to the prior of the more confident player μ_R. ■

Proof of Proposition 9.2.

Let $\gamma_S^2 \neq \gamma_R^2$ and $\mu_S = \mu_R = \mu$. Recall from Step 1 in the proof of Proposition 9.1.b) that $\Delta(\sigma)$ is symmetrically V-shaped around $\widetilde{\sigma}$ while $\Delta(\widetilde{\sigma}) = 0$. This immediately implies that any signal weakly increases disagreement relative to the prior disagreement (of 0).

We first show that there exists an equilibrium $\widetilde{D}$ where all signals besides $\widetilde{\sigma} = \mu$ are not disclosed, while $\widetilde{\sigma}$ is disclosed with an arbitrary probability in $[0,1]$. Indeed, in this case the posterior disagreement conditional on no disclosure $\Delta^{\widetilde{D}}(\varnothing)$ is equal to 0, since the posterior means of both S's and R's belief distributions in the eyes of R are then equal to the prior mean μ. Hence, given the shape of $\Delta(\sigma)$, S indeed strictly prefers non-disclosure over disclosure for any σ except for $\widetilde{\sigma}$, where he is indifferent.

Let us show that no other equilibrium exists. Assume by contradiction that there exists an equilibrium disclosure rule D' such that some other signals besides $\widetilde{\sigma}$ are disclosed with positive probability. Then, $\Delta^{D'}(\varnothing)$ must be strictly positive as otherwise S would strictly prefer to conceal all signals other than $\widetilde{\sigma}$. But if $\Delta^{D'}(\varnothing) > 0$, then every signal σ such that $\Delta(\sigma) \leq \Delta^{D'}(\varnothing)$ will be disclosed and every signal such that $\Delta(\sigma) > \Delta^{D'}(\varnothing)$ will not be disclosed. Given the symmetric V-shape of $\Delta(\sigma)$, S must disclose signals belonging to an interval $[\underline{\sigma}, \overline{\sigma}]$ which is symmetric around $\widetilde{\sigma}$. Then,

$$\Delta^{D'}(\varnothing) = \left| \tau(\mu_S - \mu_R) + (1-\tau) \begin{bmatrix} \int_{\sigma \leq \underline{\sigma}} D(\sigma) \widetilde{f}(\sigma \,| d = \varnothing, \underline{\sigma}, \overline{\sigma}) d\sigma \\ + \int_{\sigma \geq \overline{\sigma}} D(\sigma) \widetilde{f}(\sigma \,| d = \varnothing, \underline{\sigma}, \overline{\sigma}) d\sigma \end{bmatrix} \right|,$$

where $D(\sigma)$ is given by (63), $\tau = P(\sigma = \varnothing \,| d = \varnothing, \underline{\sigma}, \overline{\sigma})$, and $\widetilde{f}(\sigma \,| d = \varnothing, \underline{\sigma}, \overline{\sigma})$ is the conditional distribution of σ given $d = \varnothing$ from the perspective of R. Given $\mu_S = \mu_R$ this further simplifies to

$$\Delta^{D'}(\varnothing) = \left| (1-\tau) \left[\int_{\sigma \leq \underline{\sigma}} Q(\sigma) d\sigma + \int_{\sigma \geq \overline{\sigma}} Q(\sigma) d\sigma \right] \right|, \tag{74}$$

where $Q(\sigma) = D(\sigma) \frac{f(\sigma)}{P(\sigma \notin [\underline{\sigma}(x), \overline{\sigma}(x)])}$. Note furthermore that given linearity of $D(\sigma)$, we have $D(\widetilde{\sigma} + z) = -D(\widetilde{\sigma} - z)$. Besides, by (64), we have $\widetilde{\sigma} = \mu$ and hence $f(\widetilde{\sigma} + z) = f(\widetilde{\sigma} - z)$. Thus, $Q(\widetilde{\sigma} + z) = -Q(\widetilde{\sigma} - z)$, which by the symmetry of disclosure interval around $\widetilde{\sigma}$ yields

$$\int_{\sigma \leq \underline{\sigma}} Q(\sigma) d\sigma + \int_{\sigma \geq \overline{\sigma}} Q(\sigma) d\sigma = 0.$$

This together with (74) implies $\Delta^{D'}(\varnothing) = 0$, which is a contradiction. Thus, there exists no equilibrium where any signal besides for $\widetilde{\sigma}$ is disclosed. ■

Proof of Proposition 9.1.

Step 1. If $\gamma_S^2 = \gamma_R^2$ and $\mu_S \neq \mu_R$, then

$$\begin{aligned}\Delta(\sigma) &= \left|E_S\left[\omega\left|\sigma\right.\right] - E_R\left[\omega\left|\sigma\right.\right]\right| = \left|\frac{\gamma_\varepsilon^2}{\gamma_R^2+\gamma_\varepsilon^2}\left(\mu_S-\mu_R\right)\right| \\ &< \left|\mu_S-\mu_R\right|. \end{aligned} \tag{75}$$

Consequently, there exists an equilibrium with full disclosure. Let us show that no other equilibrium exists. Assume by contradiction that there exists an equilibrium featuring a non-empty disclosure interval $\widetilde{I}$. Then,

$$\begin{aligned} &\Delta^{\widetilde{I}}(\varnothing) \\ &= \left|\begin{array}{c} P(\sigma=\varnothing\left|d=\varnothing,\underline{\sigma},\overline{\sigma}\right.)(\mu_S-\mu_R) \\ +P(\sigma\neq\varnothing\left|d=\varnothing,\underline{\sigma},\overline{\sigma}\right.)\int_{\sigma\in\widetilde{I}}D(\sigma)\widetilde{f}(\sigma\left|d=\varnothing\right.)d\sigma \end{array}\right| \\ &= \left|\begin{array}{c} P(\sigma=\varnothing\left|d=\varnothing,\underline{\sigma},\overline{\sigma}\right.)(\mu_S-\mu_R) \\ +P(\sigma\neq\varnothing\left|d=\varnothing,\underline{\sigma},\overline{\sigma}\right.)\left(\frac{\gamma_\varepsilon^2}{\gamma_R^2+\gamma_\varepsilon^2}\left(\mu_S-\mu_R\right)\right) \end{array}\right| \\ &> \left|\frac{\gamma_\varepsilon^2}{\gamma_R^2+\gamma_\varepsilon^2}\left(\mu_S-\mu_R\right)\right| = \Delta(\sigma) \end{aligned}$$

for any σ. Hence, S would have incentive to deviate to disclosure for any $\sigma \in \widetilde{I}$.

Step 2. Let $\gamma_S^2 = \gamma_R^2$ and $\mu_S = \mu_R$, then by (63) $D(\sigma) = \Delta(\sigma) = 0$ for any σ. It follows immediately that any disclosure rule is an equilibrium disclosure rule. ■

Appendix IX: Propositions 10 and 11

Proof of Proposition 10

Step 1. Consider the disclosure choice of the agent (S) in a putative FD-equilibrium if he holds a signal $\sigma \in \{0, 1\}$. Recall that S prefers a higher $\Pi^{FD}(d)$, since he maximizes the probability of being hired. Hence, in an FD-equilibrium, S has no strict incentive to deviate when holding a σ-signal if and only if $\Upsilon(\sigma, \beta_S, \beta_R) \geq 0$ for $\sigma \in \{0, 1\}$. We have

$$\begin{aligned}
\Upsilon(1, \beta_S, \beta_R) &= \Pi^{FD}(1) - \Pi^{FD}(\varnothing) \\
&= \widetilde{\beta}_R(1)\widetilde{\beta}_S(1) + (1 - \widetilde{\beta}_R(1))(1 - \widetilde{\beta}_S(1)) \\
&\quad -\beta_R\beta_S - (1 - \beta_R)(1 - \beta_S) \\
&= \frac{(\beta_S + \beta_R - 2\beta_R\beta_S)(2p - 1)}{(1 - p + \beta_R(2p - 1))(1 - p + \beta_S(2p - 1))} \\
&\quad \times((\beta_R + \beta_S - 1)(1 - p) + \beta_R\beta_S(2p - 1)).
\end{aligned}$$

It is easy to show that the fraction on the right-hand side is always positive. Hence,

$$\begin{aligned}
\Upsilon(1, \beta_S, \beta_R) &\geq 0 \\
&\Leftrightarrow (\beta_R + \beta_S - 1)(1 - p) + \beta_R\beta_S(2p - 1) \geq 0 \\
&\Leftrightarrow \beta_S \geq \frac{(1 - p)(1 - \beta_R)}{1 - p + \beta_R(2p - 1)}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Upsilon(0, \beta_S, \beta_R) &= \Pi^{FD}(0) - \Pi^{FD}(\varnothing) \\
&= \widetilde{\beta}_R(0)\widetilde{\beta}_S(0) + (1 - \widetilde{\beta}_R(0))(1 - \widetilde{\beta}_S(0)) \\
&\quad -\beta_R\beta_S - (1 - \beta_R)(1 - \beta_S) \\
&= \frac{(\beta_S + \beta_R - 2\beta_R\beta_S)(2p - 1)}{(p - \beta_R(2p - 1))(p - \beta_S(2p - 1))} \\
&\quad \times(p(1 - \beta_R - \beta_S) + \beta_R\beta_S(2p - 1)).
\end{aligned}$$

Hence,

$$\begin{aligned}
\Upsilon(0, \beta_S, \beta_R) &\geq 0 \\
&\Leftrightarrow (p(1-\beta_R-\beta_S)+\beta_R\beta_S(2p-1)) \geq 0 \\
&\Leftrightarrow \beta_S \leq \frac{p(1-\beta_R)}{\beta_R+p(1-2\beta_R)}.
\end{aligned}$$

Thus, $\Upsilon(0, \beta_S, \beta_R)$ and $\Upsilon(1, p, \beta_S, \beta_R)$ are both positive if and only if

$$\beta_S \in \left[\frac{(1-p)(1-\beta_R)}{1-p+\beta_R(2p-1)}, \frac{p(1-\beta_R)}{\beta_R+p(1-2\beta_R)}\right].$$

This condition is identical to the one for FD appearing in Proposition 1.

Step 2. In a putative D1-equilibrium, S must find it optimal to conceal 0-signals and disclose 1-signals. This requires

$$\begin{aligned}
\Pi^{FD}(0) &\leq \Pi^{D1}(\varnothing), \\
\Pi^{FD}(1) &\geq \Pi^{D1}(\varnothing),
\end{aligned}$$

where $\Pi^{D1}(\varnothing)$ is the value of Π after no disclosure in a putative equilibrium where only 1-signals are disclosed. Note that

$$\Pi^{D1}(\varnothing) = \Pr[\sigma=0|\varnothing]\Pi^{FD}(0) + (1-\Pr[\sigma=0|\varnothing])\Pi^{FD}(\varnothing).$$

Consequently,

$$\Pi^{FD}(0) \leq \Pi^{D1}(\varnothing) \text{ iff } \Pi^{FD}(0) \leq \Pi^{FD}(\varnothing).$$

Hence, by Step 1,

$$\Pi^{FD}(0) \leq \Pi^{D1}(\varnothing) \text{ iff } \beta_S \geq \frac{p(1-\beta_R)}{\beta_R+p(1-2\beta_R)}.$$

Next, $\Pi^{FD}(0) \leq \Pi^{D1}(\varnothing)$ immediately implies the remaining condition $\Pi^{FD}(1) \geq \Pi^{D1}(\varnothing)$. Indeed,

$$\frac{p(1-\beta_R)}{\beta_R+p(1-2\beta_R)} > \frac{(1-p)(1-\beta_R)}{1-p+\beta_R(2p-1)}.$$

Consequently,

$$\begin{aligned}\beta_S &\geq \frac{p(1-\beta_R)}{\beta_R + p(1-2\beta_R)} \\ &\Rightarrow \beta_S > \frac{(1-p)(1-\beta_R)}{1-p+\beta_R(2p-1)} \\ &\Leftrightarrow \Pi^{FD}(1) > \Pi^{FD}(\varnothing),\end{aligned}$$

where the last step is by Step 1. Thus, a D1-equilibrium exists if and only if $\beta_S \geq \frac{p(1-\beta_R)}{\beta_R+p(1-2\beta_R)}$, which is equivalent to the corresponding condition in Proposition 1.

Step 3. The proof for the D0-equilibrium proceeds analogously to Step 2. ■

Proof of Proposition 11

Preliminary step. Steps 1 and 2 establish preliminary results that are used in the remaining steps. Step 1 pins down equilibrium efforts in the Tullock contest with known disagreement. Step 2 proves conditions under which full disclosure is an equilibrium outcome in the one shot disclosure game if the utility function of S and R is $-(\omega - a)^2$. Step 3 and 4 prove respectively points a) and b).

Step 1. Consider the Tullock contest with known beliefs $\widehat{\beta}_i$ and $\widehat{\beta}_j$. Denote by W_i and L_i the expected payoff of i if authority is assigned to respectively i and j. Given that the optimal action of i conditional on having authority in stage 3 is $a = \widehat{\beta}_i$, and there is no belief uncertainty by assumption, we have

$$W_i = -\left((1-\widehat{\beta}_i)(\widehat{\beta}_i)^2 + \widehat{\beta}_i(1-\widehat{\beta}_i)^2\right) = \widehat{\beta}_i\left(\widehat{\beta}_i - 1\right)$$

and

$$L_i = -\left((1-\widehat{\beta}_i)(\widehat{\beta}_j)^2 + \widehat{\beta}_i(1-\widehat{\beta}_j)^2\right).$$

It is easily checked that

$$W_i - L_i = W_j - L_j = \left(\widehat{\beta}_i - \widehat{\beta}_j\right)^2. \tag{76}$$

At the Tullock contest stage, given belief profile $\{\widehat{\beta}_i, \widehat{\beta}_j\}$, the objective function of agent i is:

$$\begin{aligned} & \left(\frac{e_i}{e_i+e_j}\right) W_i + \left(1 - \frac{e_i}{e_i+e_j}\right) L_i - \mu e_i \\ = & \left(\frac{e_i}{e_i+e_j}\right)(W_i - L_i) + L_i - \mu e_i. \end{aligned}$$

Given e_j, the FOC w.r.t. effort e_i for agent i reads:

$$\frac{(e_i+e_j) - e_i}{(e_i+e_j)^2}(W_i - L_i) - \mu = 0.$$

Letting $\Gamma = W_i - L_i$, this rewrites as $\frac{e_j \Gamma_i}{(e_i+e_j)^2} = \mu$,which yields the unique solution

$$e_i^* = -e_j + \frac{1}{\sqrt{\mu}}\sqrt{\Gamma_i}\sqrt{e_j}.$$

At the Tullock contest stage, given $\Gamma_i = \Gamma_j = \Gamma$ by (76), equilibrium efforts e_i and e_j are obtained by solving the following system of two equations and two unknowns:

$$\begin{aligned} e_i &= -e_j + \frac{1}{\sqrt{\mu}}\sqrt{\Gamma}\sqrt{e_j} \\ e_j &= -e_i + \frac{1}{\sqrt{\mu}}\sqrt{\Gamma}\sqrt{e_i}. \end{aligned}$$

The above yields the following unique solution:

$$e_i^*(\Gamma, \mu) = e_j^*(\Gamma, \mu) = \frac{\Gamma}{4\mu} = \frac{\left(\widehat{\beta}_i - \widehat{\beta}_j\right)^2}{4\mu}. \tag{77}$$

Step 2. Consider the following simple disclosure game. S holds a signal which she may disclose or not. Then R, after observing S's disclosure, chooses an action. Both players' utility function is given by $-(a-\omega)^2$. In this simple disclosure game, there exists an FD-equilibrium if and only if $\beta_S \in [I_{outcome}]$, i.e. if and only if β_S is close enough to β_R. To see this, assume such a putative equilibrium. Suppose that S

holds a 0-signal. Then S is willing to disclose if and only if

$$\begin{aligned} & \left(1-\widetilde{\beta}_S(0)\right)\left(\widetilde{\beta}_R(0)\right)^2+\widetilde{\beta}_S(0)\left(1-\widetilde{\beta}_R(0)\right)^2 \\ < \ & (1-\widetilde{\beta}_S(0))\,(\beta_R)^2+\widetilde{\beta}_S(0)(1-\beta_R)^2. \end{aligned}$$

This is equivalent to

$$\beta_S < \widehat{\beta}_S^{**}(\beta_R,p)=\frac{\beta_R p(1-\beta_R(2p-1))}{\beta_R(1-\beta_R)(1-2p)^2+2(1-p)p}.$$

Suppose that S holds a 1-signal. Then S is willing to disclose if and only if

$$\begin{aligned} & \left(1-\widetilde{\beta}_S(1)\right)\left(\widetilde{\beta}_R(1)\right)^2+\widetilde{\beta}_S(1)\left(1-\widetilde{\beta}_R(1)\right)^2 \\ < \ & (1-\widetilde{\beta}_S(1))\,(\beta_R)^2+\widetilde{\beta}_S(1)(1-\beta_R)^2. \end{aligned}$$

This is equivalent to

$$\beta_S > \widehat{\beta}_S^{*}(\beta_R,p)=\frac{\beta_R(1-p)(1+\beta_R(2p-1))}{\beta_R(1-\beta_R)(1-2p)^2+2(1-p)p}.$$

Step 3. This proves point a). Suppose by contradiction that β_S does not belong to $I_{disagreement}$ and that equilibrium features full disclosure in stage 1. Given $\beta_S \notin I_{disagreement}$, there is some signal realization σ such that disclosing (as opposed to not disclosing) strictly increases R's perception of disagreement in stage 1 from $|\beta_S-\beta_R|$ to $\left|\widetilde{\beta}_S(\sigma)-\widetilde{\beta}_R(\sigma)\right| > |\beta_S-\beta_R|$. For such σ, by Step 1 disclosing yields opponent's effort $\frac{\left(\widetilde{\beta}_S(\sigma)-\widetilde{\beta}_R(\sigma)\right)^2}{4\mu}$ whereas not disclosing yields the strictly lower opponent's effort $\frac{(\beta_S-\beta_R)^2}{4\mu}$. Note that a strictly smaller effort of R in stage 2 is always strictly advantageous for S, as this strictly increases S's probability of advantageously being assigned authority for any given e_S. Note also that if not disclosing in stage 1, S retains the option of disclosing the omitted signal in stage 3, thereby achieving the same final belief of R in stage 3 as if she had disclosed in stage 1. It follows immediately that for σ such that $\left|\widetilde{\beta}_S(\sigma)-\widetilde{\beta}_R(\sigma)\right| > |\beta_S-\beta_R|$, S has a strict incentive to deviate to non-disclosure.

Step 4. This proves point b). Under the assumed constraints on parameter values, we now show that we can construct a putative equilibrium featuring FD by S in stage 1. The equilibrium features the following strategy profile. In stage 4, R and S pick respectively $a = \widetilde{\beta}_R(d)$ and $a = \widetilde{\beta}_S(\sigma)$ if assigned authority. In stage 3 (second round of disclosure), if S did not disclose his signal then she discloses it. In stage 2, given disclosure d in stage 1, R chooses effort level $\widetilde{e}_R(d) = \frac{\left(\widetilde{\beta}_S(d) - \widetilde{\beta}_R^{FD}(d)\right)^2}{4\mu}$, where $\widetilde{\beta}_R^{FD}(\varnothing) = \beta_R$ and $\widetilde{\beta}_S(\varnothing) = \beta_S$. S instead picks

$$e_S^*(\sigma, d) = -\widetilde{e}_R(d) + \frac{1}{\sqrt{\mu}} \sqrt{\left(\widetilde{\beta}_S(\sigma) - \widetilde{\beta}_R^{FD}(\sigma)\right)^2} \sqrt{\widetilde{e}_R(d)},\ \sigma \in$$

Note that $\left(\widetilde{\beta}_S(\sigma) - \widetilde{\beta}_R^{FD}(\sigma)\right)^2$ represents the expected payoff difference, for S, between being assigned authority and not being assigned authority, anticipating future behavior of players as defined by players' equilibrium strategies. Consider finally stage 1. Here, S always discloses. Note that given anticipated future behavior, deviating from disclosing in stage 1 leaves the stage 4 action unaffected whatever the final allocation of authority. The only effect of not disclosing in stage 1 is a weak increase in R's effort in stage 2 (since $\beta_S \in I_{disagreement}$ by assumption), which is unambiguously payoff decreasing for S given anticipated future behavior. Deviating from disclosing in period 1 thus cannot be strictly advantageous. The strategies in stages 2 and 4 are optimal given beliefs and subsequent behavior. The strategy in stage 3 is optimal since $\beta_S \in [I_{outcome}]$ by assumption. ■

Appendix X: Propositions 12 and 13

Proof of Proposition 12

Step 1. This proves Point 1 of the proposition. Assume without loss of generality that $x \geq y$. Given the definitions in section 3.4 we have

$$\begin{aligned}
&\Lambda^x(x,y,p) \\
= \;& P(\sigma = 1\,|x)D_1(x,y,p) + P(\sigma = 0\,|x)D_0(x,y,p) \\
= \;& (xp + (1-x)(1-p))\left(\frac{xp}{xp + (1-x)(1-p)} - \frac{yp}{yp + (1-y)(1-p)}\right) \\
& + (1 - (xp + (1-x)(1-p)))\left(\frac{x(1-p)}{x(1-p) + (1-x)p} - \frac{y(1-p)}{y(1-p) + (1-y)p}\right).
\end{aligned}$$

At the same time,

$$\Lambda^x(x,y,\frac{1}{2}) = x - y.$$

The difference $V^x(x,y,p) = \Lambda^x(x,y,\frac{1}{2}) - \Lambda^x(x,y,p)$ further simplifies to

$$V^x(x,y,p) = \frac{(1-2p)^2(x-y)(1-y)y}{(y+p-2py)(2py+1-(p+y))}. \tag{78}$$

It can be trivially shown that this expression is always positive no matter the values of x, y and p, where V^x equals to 0 if and only if $y \in \{0, x, 1\}$, which proves Point 1 of the proposition.

Step 2. Let us show that the derivative of $V^x(x,y,p)$ with respect to y is convex in y if $x > y$ and concave in y if $y > x$. Consider $x > y$. Taking the third derivative of $V^x(x,y,p)$ and simplifying we obtain

$$\frac{\partial^3 V^x(x,y,p)}{\partial y^3} = \frac{6(1-2p)^2(1-p)p}{(1-y-p(1-2y))^4(y+p(1-2y))^4}M, \tag{79}$$

where

$$
\begin{aligned}
M &= y^4(1-2p)^4 - 4y^3x(1-2p)^4 + p - 4p^2 + 6p^3 - 3p^4 + 6y^2(x(1-2p)^4 \\
&\quad +(1-2p)^2(1-p)p) + x(1-2p)^2(1-2p+2p^2) \\
&\quad -4y(1-2p)^2(x+p-3xp-p^2+3xp^2).
\end{aligned}
$$

Let us show that $M > 0$. Note that M is linear in x. Hence, to prove that M as a function of x is positive on $(y, 1)$ it is sufficient to show that it is positive at the boundaries of this interval. We have that at $x = y$

$$
\begin{aligned}
M_{|x=y} &= 6y^3(1-2p)^4 - 3y^4(1-2p)^4 + p - 4p^2 + 6p^3 - 3p^4 \\
&\quad +y(1-2p)^2(1-6p+6p^2) - 2y^2(1-2p)^2(2-9p+9p^2).
\end{aligned}
$$

One can verify that this function of y has no roots on [0,1]. Besides at $y = 0$ this expression turns to $p(1-4p) + 6p^3(1-0.5p) > 0$. Hence,

$$
M_{|x=y} > 0. \tag{80}
$$

Next,

$$
\begin{aligned}
M_{|x=1} &= 1 - 4y^3(1-2p)^4 + y^4(1-2p)^4 - 5p + 10p^2 - 10p^3 + 5p^4 \\
&\quad -4y(1-2p)^2(1-2p+2p^2) + 6y^2(1-2p)^2(1-3p+3p^2).
\end{aligned}
$$

One can verify that this function of y has no roots on [0,1]. Besides at $y = 0$ this expression turns to $1 - 5p(1-2p) - 10p^3(1-0.5p) > 0$. Hence,

$$
M_{|x=1} > 0.
$$

This together with (80) and the fact that M is linear in x implies that $M > 0$. Consequently, by (79)

$$
\frac{\partial^3 V^x(x,y,p)}{\partial y^3} > 0,
$$

i.e. the derivative of V^x with respect to y is convex in y if $x > y$.

The claim that the derivative of V^x with respect to y is concave in y if $y > x$ follows analogously.

Step 3. Now we can prove Point 2 of the proposition. From Step 1 and the continuity of $V^x(x, y, p)$ in y it follows that

$$\begin{aligned} \frac{\partial V^x(x,y,p)}{\partial y}_{|y=0} &> 0, \\ \frac{\partial V^x(x,y,p)}{\partial y}_{|y \to x^-} &< 0, \end{aligned}$$

Since further $\frac{\partial V^x(x,y,p)}{\partial y}$ is convex in y by Step 2, it follows that it has a unique root on $(0, x)$. This implies that $V^x(x, y, p)$ is single-peaked in y for $y \in [0, x]$. The claim for $x < y$ follows analogously given that

$$\begin{aligned} \frac{\partial V^x(x,y,p)}{\partial y}_{|y=x^+} &> 0, \\ \frac{\partial V^x(x,y,p)}{\partial y}_{|y=1^-} &< 0 \end{aligned}$$

by Step 1, and $\frac{\partial V^x(x,y,p)}{\partial y}$ is concave in y for $x < y$ by Step 2.

Step 4. Now we can prove Point 3 of the proposition. Let us show that for $x < 1/2$ the maximum of $V^x(x, y, p)$ is reached for $y > 1/2$ (the reverse argument then immediately follows by symmetry considerations). First, note that for $x = 1/2$ we should have that the left and the right peaks (see Step 3) yield the same value of $V^x(x, y, p)$ by symmetry considerations. Next, we have that

$$y > x : \frac{\partial V^x(x,y,p)}{\partial x} = \frac{(1-y)y(1-2p)^2}{(y-1+p-2yp)(y+p-2yp)} < 0, \tag{81}$$

$$y < x : \frac{\partial V^x(x,y,p)}{\partial x} = -\frac{(1-y)y(1-2p)^2}{(y-1+p-2yp)(y+p-2yp)} > 0, \tag{82}$$

This implies that as x decreases, $\max_y V^x(x, y, p|y > x)$ is increasing and $\max_y V^x(x, y, p|y < x)$ is decreasing. Hence, overall $\max_y V^x(x, y, p)$ is reached at $\widehat{y} > x$. To show that

$\widehat{y} > 1/2$ we use the fact that

$$\begin{aligned} & \frac{\partial V^x(x,y,p)}{\partial y}_{|y=1/2} \\ = & \frac{4(1-2p)^2(-\frac{3}{16}(1-2p)^2 - x(1-p)p + (1+x)(1-p)p + \frac{1}{4}(1-7(1-p)p))}{(1-p+\frac{1}{2}(2p-1))^2} \\ > & \ 0. \end{aligned}$$

Hence, the right peak (maximizing $V^x(x, y, p)$) is reached to the right of $y = 1/2$. ■

Proof of Proposition 13

We further denote

$$\mu(x, y) = \min\{V^x(x, y), V^y(x, y)\}.$$

Given that both players should agree to participate, the probability of signal acquisition is maximized if and only if $\mu(x, y)$ is maximized.

Step 1. Note that $\mu(x, y)$ should reach its maximum at some values $\{x^*, y^*\}$ where $x^*, y^* \neq \{0, 1\}$ since $\min\{V^x(x, y), V^y(x, y)\} = 0$ if either x or y are at the boundaries while there exists some $\{x, y\}$ where $\mu(x, y) > 0$ (by Proposition 12.1).

Step 2. By (81) and (82) we have that $V^x(x, y)$ is linearly increasing (decreasing) in x for $x > y$ ($x < y$). Analogously, $V^y(x, y)$ is linearly increasing (decreasing) in y for $y > x$ ($y < x$).

Step 3. Let us show that $\mu(x, y)$ must reach its maximum at some $\{x^*, y^*\}$ where $V^x(x^*, y^*) = V^y(x^*, y^*)$. Assume by contradiction that this is not the case so that for instance, $V^x(x^*, y^*) < V^y(x^*, y^*)$. Then, by Steps 1 and 2 one can slightly change x to raise the value of $V^x(x, y)$ so that $\mu(x, y) = V^x(x, y) < V^y(x, y)$ continues to hold. In other words, one can raise $\mu(x, y)$ at least by a slight perturbation of x which proves that $\{x^*, y^*\}$ is not the optimum. The symmetric argument excludes $V^x(x^*, y^*) > V^y(x^*, y^*)$.

Step 4. We have shown that $V^x(x^*, y^*) = V^y(x^*, y^*)$. Given (78) (and the symmetrical expression for $V^y(x, y, p)$), this condition holds if and only if either $x^* =$

y^* or $D_0(x^*, y^*) = D_1(x^*, y^*)$ (in which case a difference in the probability weights on $D_0(x^*, y^*)$ and $D_1(x^*, y^*)$ does not matter). One can in turn verify that the latter condition is true if and only if either $x^* = y^*$ or $x^* = 1 - y^*$. In the first case, we have $\mu(x, x) = 0$ by Proposition 12.1 so it cannot be optimal. Hence, at the optimum it must hold $x^* = 1 - y^*$.

Step 5. Let us finally show that there is unique $x^* \in (0, 1/2)$ where $\mu(x^*, 1 - x^*)$ is maximized (in which case $\mu(x, y)$ is also maximized by Step 4). Let us show that $\mu(x, 1 - x)$ is concave. Note that by symmetry considerations $V^x(x, 1 - x) = V^{1-x}(x, 1 - x)$. Hence,

$$
\begin{aligned}
& \frac{\partial^2 \mu(x, 1-x)}{\partial x^2} \\
= \; & \frac{\partial^2 V^x(x, 1-x)}{\partial x^2} \\
= \; & 2(1-p)p(2p-1)\left(\frac{1}{(p(1-2x)+x-1)^3} + \frac{1}{(p(1-2x)+x)^3}\right) < 0.
\end{aligned}
$$

Given that $\mu(x, 1 - x)$ is concave in x and is equal to 0 at $x = 0$ and $x = 1/2$ by Proposition 12.1, we obtain that there is unique $x^* \in (0, 1/2)$ maximizing $\mu(x, 1 - x)$.

■